The Three Point Bend Test

1 Beam theory

The three point bend test (Figure 1) is a classical experiment in mechanics, used to measure the Young’s modulus of a material in the shape of a beam. The beam, of length $L$, rests on two roller supports and is subject to a concentrated load $P$ at its centre.

![Diagram of three point bend test](http://commons.wikimedia.org/wiki/File:SimpSuppBeamPointLoad.svg)

It can be shown (see, for example, the Cambridge University Engineering Department Structures Data Book) that the deflection $w_0$ at the centre of the beam is

$$w_0 = \frac{PL^3}{48EI} \quad (1)$$

where $E$ is the Young’s modulus. $I$ is the second moment of area defined by

$$I = \frac{a^3b}{12} \quad (2)$$

where $a$ is the beam’s depth and $b$ is the beam’s width. By measuring the central deflection $w_0$ and the applied force $P$, and knowing the geometry of the beam and the experimental apparatus, it is possible to calculate the Young’s modulus of the material.
2 Force-displacement graph

If the applied force $P$ is plotted against central displacement $w_0$, a straight line is obtained provided we remain within the elastic limit of the material (i.e. the beam returns to its original shape after deflection). The gradient of this line is

$$\frac{dP}{dw_0} = \frac{48EI}{L^3} \quad (3)$$

There are some benefits to using equation (3) instead of equation (1) for estimating $E$. We can take several measurements of $P$ and $w_0$, and deal sensibly with experimental error by finding a line of best fit from which we obtain the gradient $dP/dw_0$. There is also less need for calibration, since we only need to know changes in the measured values, not the actual values.

3 Lego implementation

In the Lego implementation, a spring is used to measure the force applied to the beam. By Hooke’s law, we know that

$$P = kL_s \quad (4)$$

where $k$ is the spring constant and $L_s$ is the extension of the spring.

![Figure 2: Lego implementation](image)

Consider now Figure 2. With the Lego kit, it is relatively straightforward to measure the position of the bottom of the spring for different beam deflections. The change in force between the scenario on the left and the one on the right is

$$dP = kdL_s = k(L_{s2} - L_{s1}) = k(D - dw_0) \quad (5)$$

where $D$ is the distance travelled by the bottom of the spring. $dP$ can then be plotted against $dw_0$, with the gradient $dP/dw_0$ obtained from the line of best fit.