The Three Point Bend Test

1 Beam theory

The three point bend test (Figure 1) is a classical experiment in mechanics, used to measure the Young's modulus of a material in the shape of a beam. The beam, of length L, rests on two roller supports and is subject to a concentrated load P at its centre.

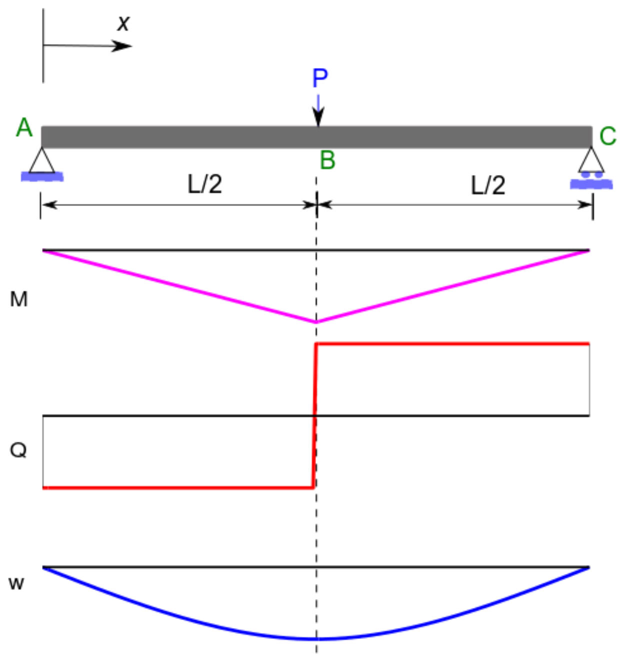

Figure 1: Schematic of the three point bend test (top), with graphs of bending moment M, shear Q and deflection w. Figure reproduced from http://commons.wikimedia.org/wiki/File:SimpSuppBeamPointLoad.svg.

It can be shown (see, for example, the *Cambridge University Engineering Department Structures Data Book*) that the deflection w_0 at the centre of the beam is

$$w_0 = \frac{PL^3}{48EI} \tag{1}$$

where E is the Young's modulus. I is the second moment of area defined by

$$I = \frac{a^3 b}{12} \tag{2}$$

where a is the beam's depth and b is the beam's width. By measuring the central deflection w_0 and the applied force P, and knowing the geometry of the beam and the experimental apparatus, it is possible to calculate the Young's modulus of the material.

2 Force-displacement graph

If the applied force P is plotted against central displacement w_0, a straight line is obtained provided we remain within the elastic limit of the material (i.e. the beam returns to its original shape after deflection). The gradient of this line is

$$\frac{dP}{dw_0} = \frac{48EI}{L^3} \tag{3}$$

There are some benefits to using equation (3) instead of equation (1) for estimating E. We can take several measurements of P and w_0, and deal sensibly with experimental error by finding a line of best fit from which we obtain the gradient dP/dw_0. There is also less need for calibration, since we only need to know *changes* in the measured values, not the *actual* values.

3 Lego implementation

In the Lego implementation, a spring is used to measure the force applied to the beam. By Hooke's law, we know that

$$P = kL_s \tag{4}$$

where k is the spring constant and L_s is the extension of the spring.

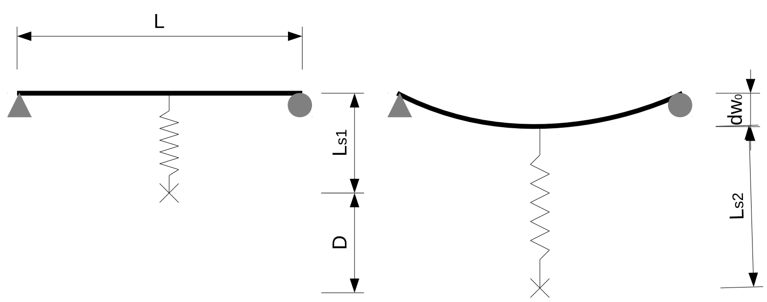

Figure 2: Lego implementation

Consider now Figure 2. With the Lego kit, it is relatively straightforward to measure the position of the *bottom* of the spring for different beam deflections. The change in force between the scenario on the left and the one on the right is

$$dP = k\, dL_s = k(L_{s2} - L_{s1}) = k(D - dw_0) \tag{5}$$

where D is the distanced travelled by the bottom of the spring. dP can then be plotted against dw_0, with the gradient dP/dw_0 obtained from the line of best fit.