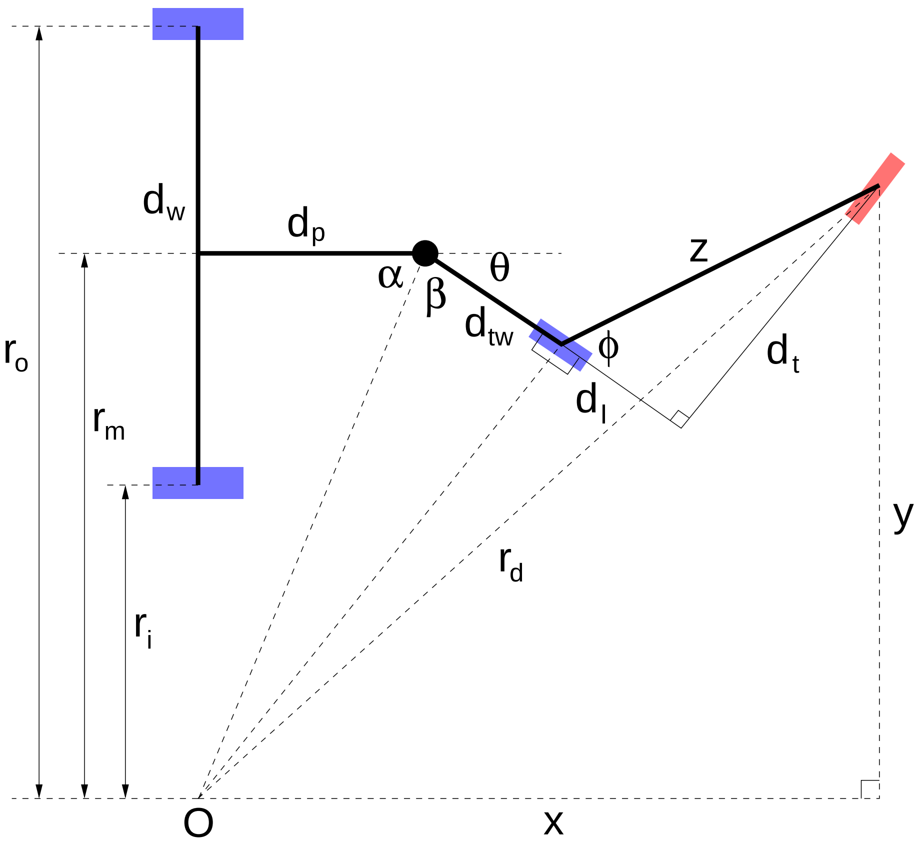
Kinematics of the Domino Layer



The geometrical properties of the domino laying vehicle are shown in the diagram above. The drive and trolley wheels are drawn in blue, with the deposited domino in red. Our aim is to lay the dominoes in a circle of radius r_d around the origin O. To do this, we need to find the radii r_i and r_o traced out by the inner and outer drive wheels. We can then set the speeds of the two drive motors according to the ratio r_i/r_o.

We start by expressing the x and y coordinates of the domino in terms of the lengths d_p, d_{tw} and z, and the radius r_m traced out by the midpoint of the front wheels:

$$x = d_p + d_{tw} \cos\theta + z\cos(\phi - \theta) \tag{1}$$

$$y = r_m - d_{tw} \sin\theta + z\sin(\phi - \theta) \tag{2}$$

$$r_d = \sqrt{x^2 + y^2} \tag{3}$$

d_p and d_{tw} can be measured directly on the robot, while ϕ and z follow trivially from the measurements d_l and d_t:

$$\phi = \tan^{-1}(d_t/d_l) \tag{4}$$

$$z = \sqrt{d_l^2 + d_t^2} \tag{5}$$

That leaves just θ. Inspecting the diagram, and noting that the trolley wheel, like the drive wheels, is travelling in a circle around O (hence the $90°$ annotations at the trolley wheel), we see that:

$$\theta = \pi - \alpha - \beta \tag{6}$$

$$\alpha = \tan^{-1}(r_m/d_p) \tag{7}$$

$$\beta = \cos^{-1}\left(d_{tw}/\sqrt{r_m^2 + d_p^2}\right) \tag{8}$$

$$\Rightarrow \theta = \pi - \tan^{-1}(r_m/d_p) - \cos^{-1}\left(d_{tw}/\sqrt{r_m^2 + d_p^2}\right) \tag{9}$$

This fully defines the *forward* kinematics. Starting from the the robot dimensions (d_p, d_{tw}, d_t and d_l), and the front wheel midpoint radius r_m, we can calculate the deposited domino radius r_d using equations (9), (1), (2) and (3), in that order.

Solving the *inverse* kinematics — i.e. starting from r_d and arriving at r_m — is much more difficult, as is often the case in real world mechanics. Rather than attempt a closed form solution, we instead provide a Matlab/Octave program that finds r_m numerically, using the forward kinematics to try different values of r_m until we arrive at one that gives the desired r_d. Once we have r_m, the required front wheel speeds follow trivially from the measured distance d_w between the front wheels:

$$r_i = r_m - d_w/2 \tag{10}$$

$$r_o = r_m + d_w/2 \tag{11}$$

$$\text{WheelSpeedRatio} = r_i/r_o \tag{12}$$