

# fMPE & pMPE – A Discriminative Semi-parametric Trajectory Model

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## Outline

- Trajectory models for speech recognition
- A *semi-parametric* trajectory model
  - trajectory mean – fMPE
  - trajectory variance – pMPE
- Discriminative training – Minimum Phone Error (MPE)
- Experimental results on Conversational Telephone Speech (CTS) tasks
- Summary

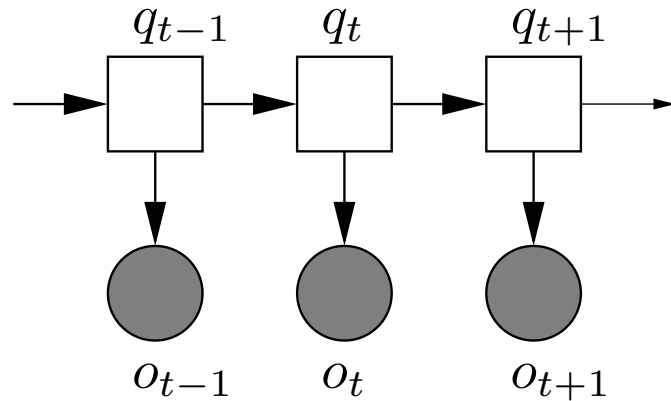


## Motivation

- HMMs – commonly used for speech recognition
- Limitations of HMMs:
  - Conditional independence assumption of observations
  - Instantaneous state transition
  - Poor duration modelling
- Conditional independence assumption implies:
  - *constant* state output probability
- Ways to overcome this problem:
  - trajectory models (e.g. buried Markov Model, trajectory HMMs)
  - segment models (e.g. stochastic segment models, segmental HMMs)
  - switching linear dynamical systems

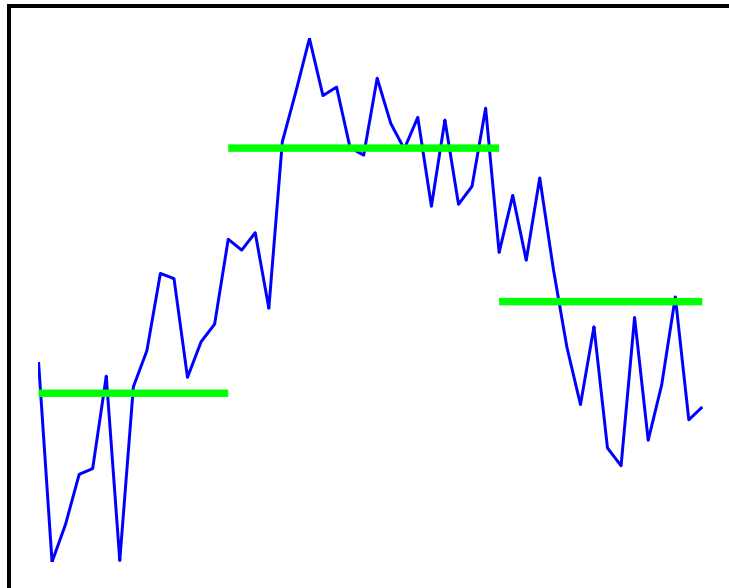


## Standard HMM



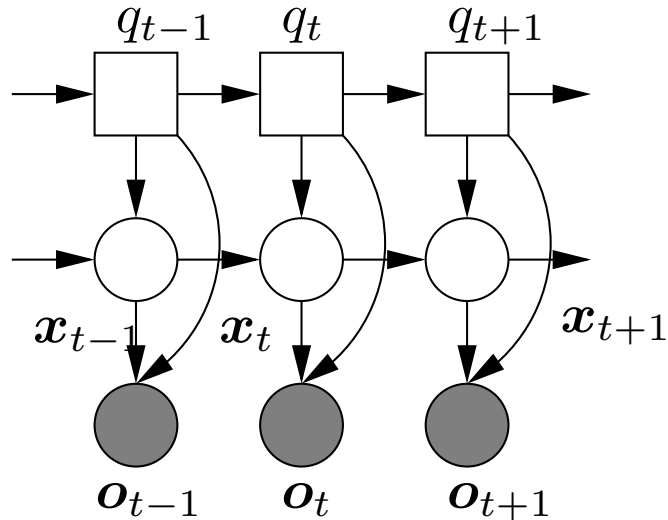
$$q_t \sim P(q_t | q_{t-1})$$

$$o_t \sim \mathcal{N}(o_t; \mu_{q_t}, \Sigma_{q_t})$$



- Conditional independence assumption
- *Piece-wise constant* trajectory within state
- Poor trajectory modelling via HMM state transition

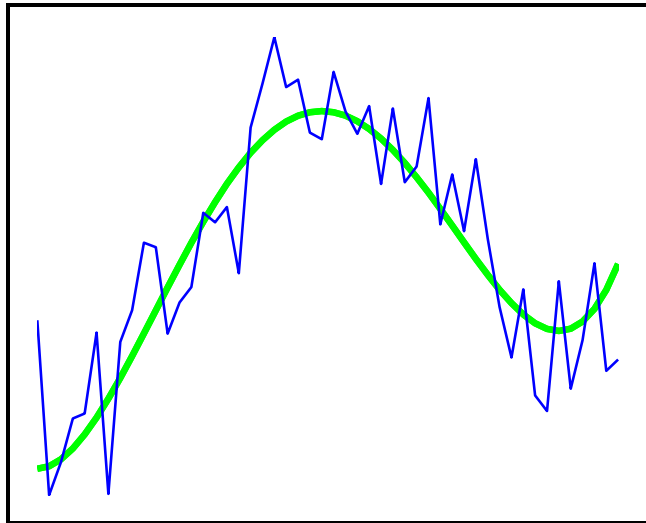
## Switching Linear Dynamical System



$$q_t \sim P(q_t|q_{t-1})$$

$$\mathbf{x}_t = \mathbf{A}_{q_t}\mathbf{x}_{t-1} + \mathbf{u}_{q_t}$$

$$\mathbf{o}_t = \mathbf{C}_{q_t}\mathbf{x}_t + \mathbf{v}_{q_t}$$



- A state-space formulation
- Model smoothed trajectory via latent variables,  $\mathbf{x}_t$
- *Time varying* mean:  $\boldsymbol{\mu}_t = \mathbf{C}_{q_t}\mathbf{x}_t$

## A Generic Trajectory Model Formulation

- A generic form of trajectory model: *non-stationary* state output distribution

$$\mathbf{o}_t \sim \sum_{m=1}^M c_{mt} \mathcal{N} \left( \boldsymbol{\mu}_t^{(m)}, \boldsymbol{\Sigma}_t^{(m)} \right)$$

- Consider time varying mean and covariance matrix
- Model time variation as a function of current (and neighbouring) observations:

$$\boldsymbol{\mu}_t^{(m)} = f \left( \boldsymbol{\mu}^{(m)}, \mathcal{M}, \mathbf{o}_1, \dots, \mathbf{o}_T \right)$$

$$\boldsymbol{\Sigma}_t^{(m)} = g \left( \boldsymbol{\Sigma}^{(m)}, \mathcal{M}, \mathbf{o}_1, \dots, \mathbf{o}_T \right)$$

- What form should  $f(\cdot)$  and  $g(\cdot)$  take?
- How many frames around  $\mathbf{o}_t$  should be considered?



## Trajectory Mean – fMPE

- Apply a time dependent bias,  $b_{tj}$ , to  $j$ th element of mean

$$\mu_{tj}^{(m)} = f(\mu_j^{(m)}, b_j^{(i)}, \mathbf{o}_t) = \mu_j^{(m)} + b_{tj}$$

- Weighted interpolation of a set of bias vectors,  $b_j^{(i)}$

$$b_{tj} = \sum_{i=1}^n h_{ti} b_j^{(i)} \quad \rightarrow \quad h_{ti} : \text{time varying interpolation weights}$$

- Equivalent to fMPE (Povey .et .al ICASSP 2005):

$$\hat{\mathbf{o}}_t = \mathbf{o}_t - \mathbf{M}\mathbf{h}_t \quad \text{where} \quad \mathbf{M} = \begin{bmatrix} \vdots & & \vdots \\ b_j^{(1)} & \dots & b_j^{(n)} \\ \vdots & & \vdots \end{bmatrix} \quad \text{and} \quad \mathbf{h}_t = \begin{bmatrix} h_{t1} \\ h_{t2} \\ \vdots \\ h^{tn} \end{bmatrix}$$



## Trajectory Covariance Matrix – pMPE

- Assume diagonal covariance matrix
- Apply a time varying *positive* scale factor,  $a_{tj}$ , to  $j$ th variance element

$$\sigma_{tj}^{2(m)} = g(\mu_j^{(m)}, a_j^{(i)}, \mathbf{o}_t) = \sigma_j^{2(m)} / a_{tj}$$

- Again, use weighted contribution:

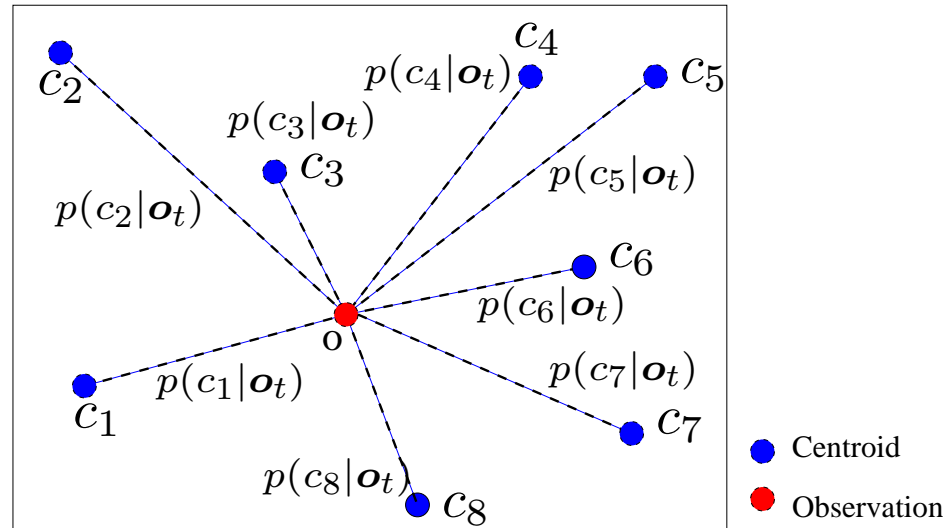
$$a_{tj} = \left( 1 + \sum_{i=1}^n h_{ti} a_j^{(i)} \right)^2 \quad \rightarrow \quad \text{taking squared to ensure positive scale}$$





## A Semi-parametric Representation

- Represent acoustic space with many *centroids*:

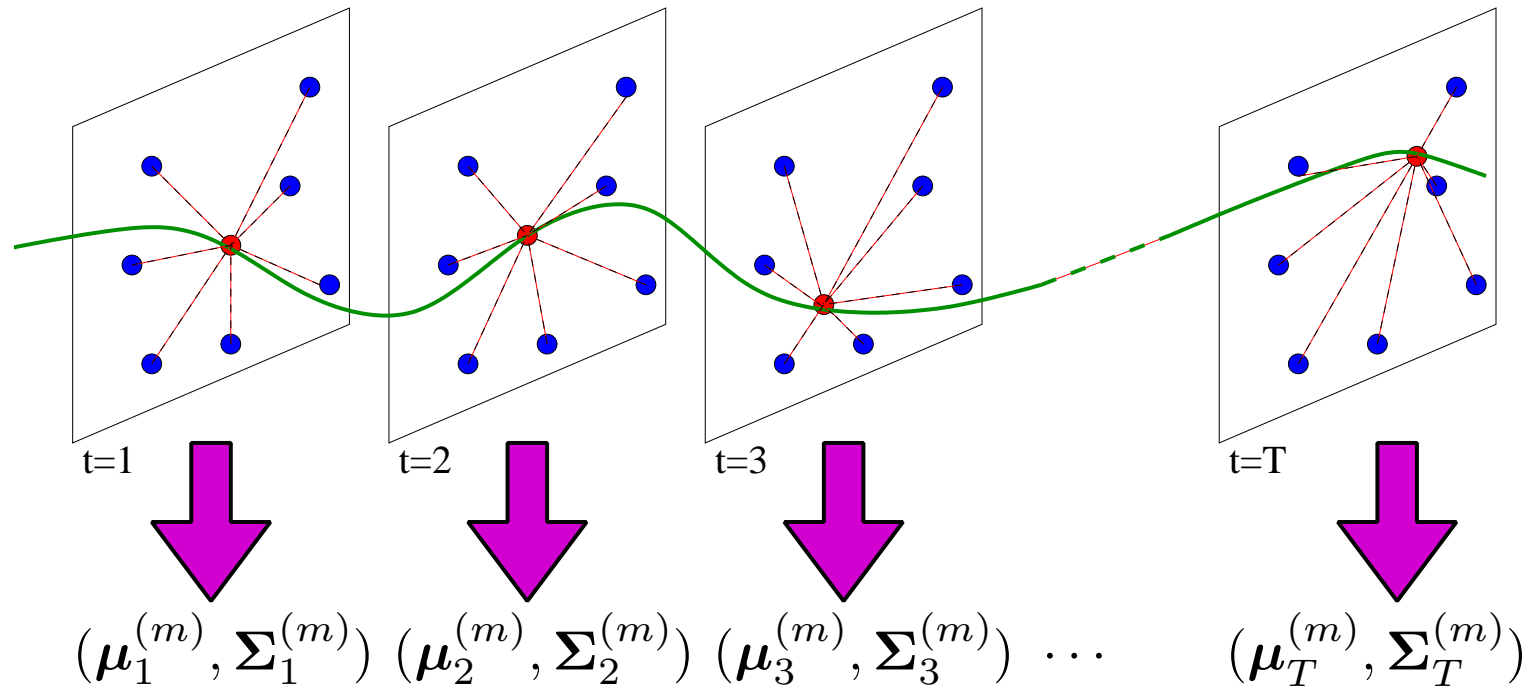


- Use centroid *posteriors* given  $\mathbf{o}_t$  as weights:

$$h_{ti} = P(c_i | \mathbf{o}_t) = \frac{\mathcal{N}(\mathbf{o}_t; c_i)}{\sum_{j=1}^n \mathcal{N}(\mathbf{o}_t; c_j)}$$

- Parameters to be trained for each centroid: bias  $b_j^{(i)}$  & scale  $a_j^{(i)}$

## Overall Smoothed Trajectory Parameters



- Overall trajectory – *weighted* contribution from each centroid
- Effective mean bias and precision scaling – smoothed over *centroids* and *time*
- May also include contributions from preceding and succeeding observations

## Contexts Expansion

- Consider contributions from  $C$  observations on either sides ( $2C + 1$  frames)

$$b_{tj} = \sum_{i=1}^n \sum_{\tau=-C}^C w_i(t - \tau) b_{\tau j}^{(i)} \quad \text{and} \quad a_{tj} = \left( 1 + \sum_{i=1}^n \sum_{\tau=-C}^C w_i(t - \tau) a_{\tau j}^{(i)} \right)^2$$

$$w_i(t - \tau) = \begin{cases} h_{ti} & \tau = 0 \\ h_{(t-\tau)i}/2 & \tau = \pm 1, \pm 2 \\ h_{(t-\tau)i}/3 & \text{for } \tau = \pm 3, \pm 4, \pm 5 \\ h_{(t-\tau)i}/4 & \tau = \pm 6, \pm 7, \pm 8, \pm 9 \\ \vdots & \end{cases}$$

- For,  $C = 9$ , there are 7 biases and scales for each centroid
- Parameterisation: *static*  $(\mu_j^{(m)}, s_j^{(m)})$  & *dynamic*  $(b_{\tau j}^{(i)}, a_{\tau j}^{(i)})$



## Minimum Phone Error (MPE) Training

- A discriminative training method — good improvement on LVCSR systems
- The MPE objective function:

$$\mathcal{F} = \sum_H P(H|\mathcal{O}, \mathcal{M}) l(H, \tilde{H})$$

- $P(H|\mathcal{O}, \mathcal{M})$  – posterior of hypothesis,  $H$
- $l(H, \tilde{H})$  – loss function of  $H$  given reference,  $\tilde{H}$  (measure of phone error)
- MPE training of dynamic and static parameters together is complex
- Two gradient descent based training schemes:
  - Interleaved *dynamic-static* parameters update
  - Direct *dynamic* parameters update



## Interleaved Dynamic-Static Parameters Update

- Key element — gradient (*complete* differential):

$$\frac{d\mathcal{F}}{d[b_j^{(i)}, a_j^{(i)}]} = \frac{\partial \mathcal{F}}{\partial [b_j^{(i)}, a_j^{(i)}]} + \sum_{m=1}^M \left( \frac{\partial \mathcal{F}}{\partial \mu_j^{(m)}} \frac{\partial \mu_j^{(m)}}{\partial [b_j^{(i)}, a_j^{(i)}]} + \frac{\partial \mathcal{F}}{\partial \sigma_j^{(m)2}} \frac{\partial \sigma_j^{(m)2}}{\partial [b_j^{(i)}, a_j^{(i)}]} \right)$$

- $\mu_j^{(m)}$  and  $\sigma_j^{(m)2}$  depends on  $a_j^{(i)}$  and  $b_j^{(i)}$ 
  - to simplify update: use ML update formulae
  - if only use partial differential – gain lost after ML training
- Interleave between:
  - Dynamic parameters update – MPE (gradient-based optimisation)
  - Static parameters update – ML (simple closed-form)
  - Slow – requires 3 passes over training data



## Direct Dynamic Parameters Update

- Start from a MPE trained system:
  - Assume static parameters are well-trained
  - Gradient w.r.t. static parameters  $\approx$  zero
  - Only update the dynamic parameters
- Complete differential simplifies to a partial differential:

$$\frac{d\mathcal{F}}{d[b_j^{(i)}, a_j^{(i)}]} \approx \frac{\partial\mathcal{F}}{\partial[b_j^{(i)}, a_j^{(i)}]}$$

- Quicker to train – requires 1 pass over training data



## Implementation Issues

- Likelihood calculation (given model parameters,  $\mathcal{M}$ ):

$$p(\mathbf{o}_t | \mathcal{M}) = K - \frac{1}{2} \sum_{j=1}^d \left( \log(\sigma_j^{2(m)}) - \log(a_{jt}^2) + a_{jt}^2 \frac{(o_{jt} - b_{tj} - \mu_j^{(m)})^2}{\sigma_j^{2(m)}} \right)$$

- For fMPE:
  - No additional cost (cache shifted observation  $o_{jt} - b_{tj}$ )
- For pMPE:
  - cache  $a_{jt}^2$  and  $\sum_{j=1}^d \log(a_{jt}^2)$
  - extra  $d$  multiplications and 1 addition
- Precision scale flooring:
  - more likely to overtrain pMPE parameters
  - apply *flooring* to the scaling factor



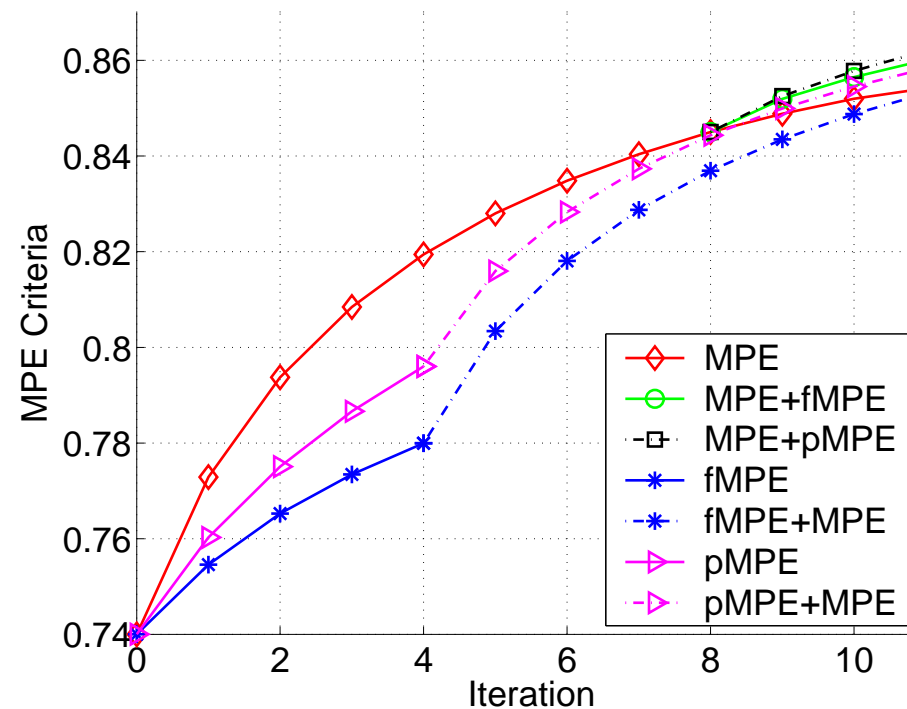
## Experimental Setup – CTS English

- Acoustic model data sets:
  - **Training data:** 297 hours
  - **Test data:** 6 hours eval103
- Front-end: PLP (static,  $C_0$ ,  $\Delta$ ,  $\Delta^2$ ,  $\Delta^3$ ) + CMN + CVN + VTLN + HLDA
- HMM acoustic models (gender independent)
- Baseline acoustic models:
  - 16 component GMMs
  - Decision tree state clustered triphones ( $\sim 6000$  states)
- Trigram language models
- $\sim 100k$  centroids without context expansion





## MPE Criterion Gain – CTS English



- Interleaved update: fMPE+MPE & pMPE+MPE
- Direct update: MPE+fMPE & MPE+pMPE
- MPE gain:  $MPE > pMPE > fMPE$
- Final systems have similar MPE criteria (0.85-0.86)



## Interleaved Update – CTS English

System	Iter 0	Iter 8
MPE	31.9	28.6
fMPE+MPE	30.1	27.8
pMPE+MPE	30.7	28.4
fMPE+pMPE+MPE	29.9	27.9

Unadapted WER performance of 16-component models on eva103

- Gains over ML: *1.8%* (fMPE) and *1.2%* (pMPE)
- Gains over MPE: *0.8%* (fMPE+MPE) and *0.2%* (pMPE+MPE)
- Combining fMPE and pMPE gave *0.2%* gain over fMPE
- Gain disappears after subsequent MPE training
  - possibly due to over training



## Direct Update – CTS English

System	Training Method	% WER
MPE	—	28.6
fMPE+MPE	Interleaved	27.8
MPE+fMPE	Direct	28.0
pMPE+MPE	Interleaved	28.4
MPE+pMPE	Direct	28.3

Unadapted WER performance of 16-component models on eva103

- Quicker to train compared to interleaved update
- Yield similar performance (slightly better for pMPE systems)
- Direct updates yield smaller gains for systems with *context expansion*
  - *partial differential* approximation not valid



## CTS English State-of-the-art Performance

System	Iter 0	Iter 8
MPE	27.5	22.8
fMPE+MPE	24.5	21.6

Unadapted WER performance of 36-component models on eval03

- Trained on approx. 2200 hours of fsh2004h5etrain03b
- Standard MPE alone — 4.7% absolute WER reduction
- fMPE (with  $C = 9$  context expansion) gain — 3.0% absolute over ML
- Overall fMPE+MPE gain — 5.9% (1.2% over standard MPE)
  - Increasing # components in standard system gives only small improvements



## Experimental Setup – CTS Mandarin

- Acoustic model data sets:
  - **Training data:** 72 hours
  - **Test data:** 2 hours dev04
- Front-end: PLP (same as English system) + pitch + Gaussianisation
- HMM acoustic models (gender independent)
- Baseline acoustic models:
  - 1 and 16 components GMMs
  - Decision tree state clustered triphones ( $\sim 4000$  states)
- Trigram language models
- $\sim 4k$  centroids with a window of 1 and 19 frames ( $C = 9$ )



## Single Component CTS Mandarin Results

System	Frames	
	1	19
MPE	44.4	44.4
fMPE+MPE	42.1	40.1
pMPE+MPE	43.3	41.3
fMPE+pMPE+MPE	41.6	38.9

Unadapted CER performance of 1-component models on dev04

- Gains over MPE:
  - 1 frame: gains for fMPE (**2.3%**) and pMPE (**1.1%**)
  - 19 frames: more gains for fMPE (**4.3%**) and pMPE (**3.1%**)
- Good improvement from fMPE+pMPE+MPE:
  - gave a further **0.5%** (1 frame) and **1.2%** (19 frames) over fMPE+MPE
- More parameters for fMPE/pMPE systems
  - *but*, only one active Gaussian component per state in decoding



## 16-component CTS Mandarin Results

System	Frames	
	1	19
MPE	36.0	36.0
fMPE+MPE	35.6	34.4
pMPE+MPE	35.9	35.4
fMPE+pMPE+MPE	35.3	34.7

Unadapted CER performance of 16-component models on dev04

- Gains over MPE:
  - 1 frame: small gains for fMPE (*0.4%*) and pMPE (*0.1%*)
  - 19 frames: larger gains for fMPE (*1.6%*) and pMPE (*0.4%*)
- Combining fMPE and pMPE:
  - 1 frame: gained further *0.3%*; 19 frames: 0.3% degradation
  - degradation may be due to over-training on limited data



## Confusion Network Combination CTS Mandarin Results

System	dev04		eval04	
	Viterbi	CN	Viterbi	CN
MPE	36.0	35.0	33.9	33.4
fMPE+MPE	34.4	33.9	32.5	32.2
fMPE+pMPE+MPE	34.7	34.0	33.1	32.6
CNC	—	33.3	—	31.6

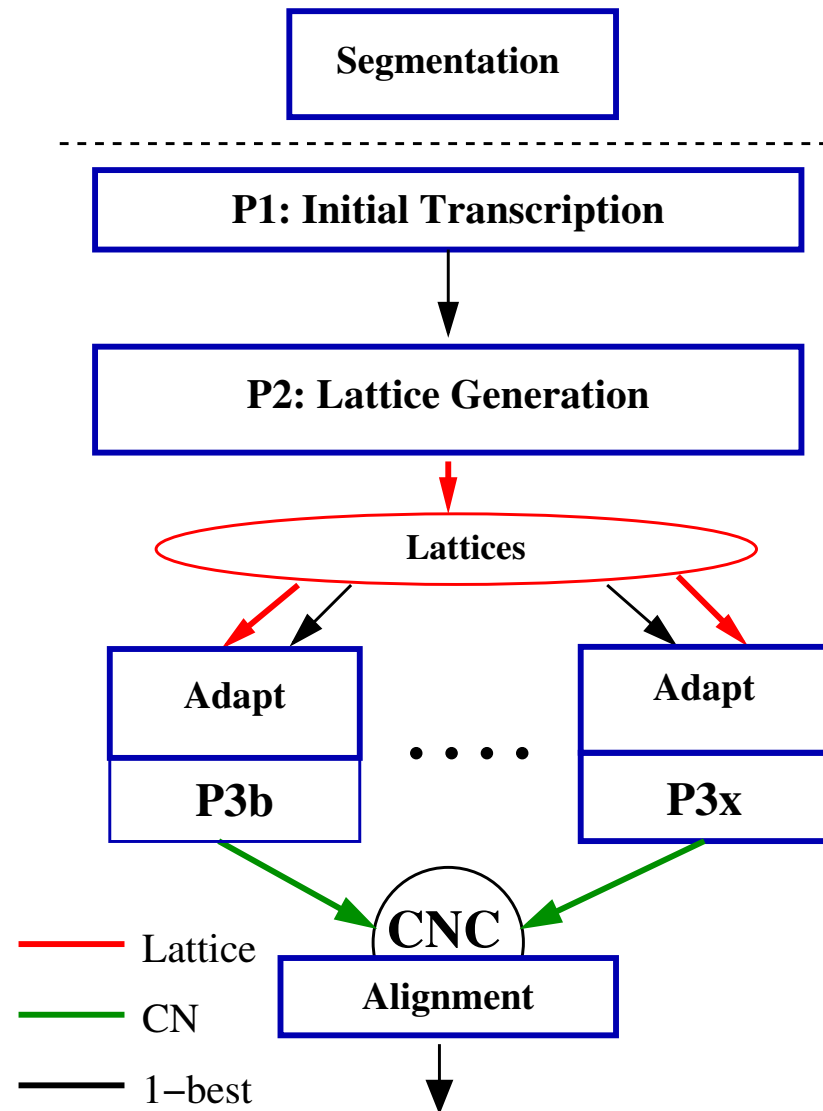
Unadapted CER performance of 16-component models on dev04 and eval04

- Using 19 frames window for fMPE and pMPE
- Confusion network (CN) decoding: *0.3%* average absolute gain
- Confusion network combination (CNC): further *0.6%* absolute improvement





# 10xRT Evaluation Framework



- Evaluation 10xRT framework:
- Multi-pass framework
- Confusion network generation
- Confusion network combination
- Adaptation in P3 stage:
  - 1-best CMLLR
  - lattice-based mean MLLR



## State-of-the-art CTS Mandarin Performance

System	CER (%)		
	HLDA	+GAUSS	+fMPE
DIAGC	35.8	34.6	33.5
+SAT	35.0	33.7	33.0
+SPAM	34.2	33.2	32.7

Adapted CN performance of various systems evaluated on dev04

- Baseline HLDA frontend with DIAGC system: (*35.8%*)
- Different acoustic models:
  - SAT: Speaker Adaptive Training
  - SPAM: an efficient precision matrix modelling scheme
- Using improved frontends:
  - Gaussianisation: gain (*1.0%*) – (*1.3%*) absolute
  - fMPE: gain (*0.5%*) – (*1.1%*) absolute
- Trade-off between frontend and acoustic model refinements.



## Summary

- Investigated semi-parametric trajectory model:
  - Trajectory mean (fMPE)
  - Trajectory variance (pMPE)
- Discriminatively trained using MPE criterion
- Gave improvement over baseline:
  - fMPE typically gave 1.0–1.5% absolute gain over MPE alone
  - Gains from pMPE smaller compared to fMPE
  - Gains of fMPE and pMPE not additive
    - \* disappears as system complexity increases
- Successfully applied on state-of-the-art CTS English and Mandarin systems

