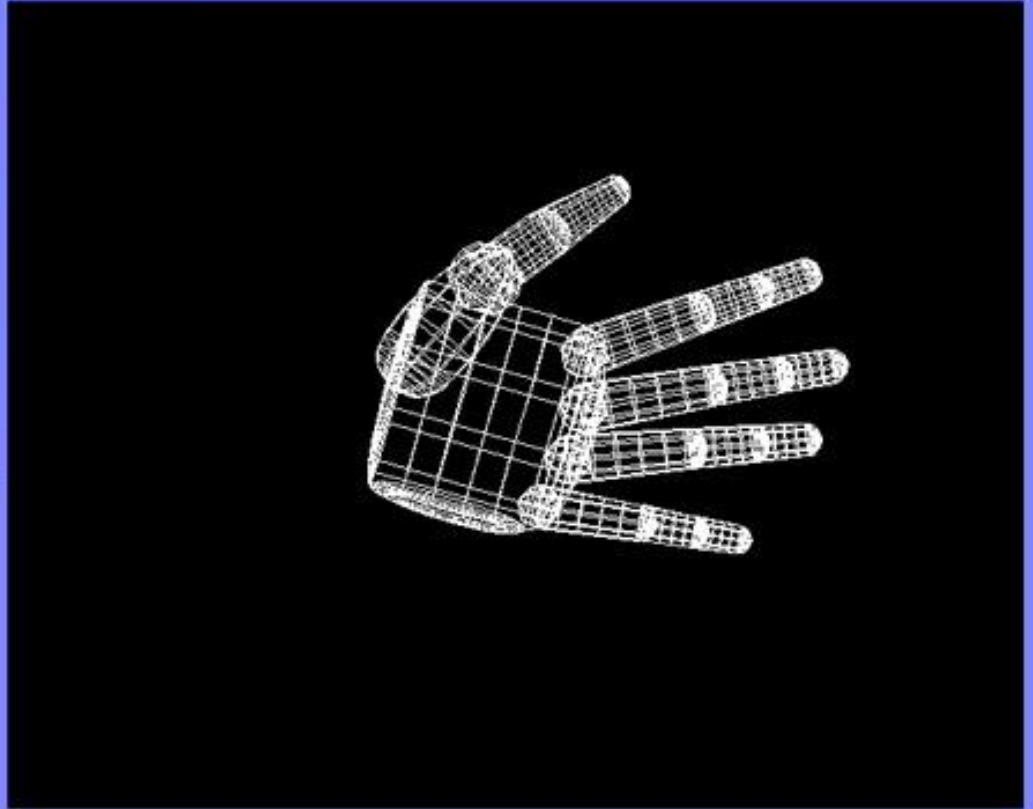
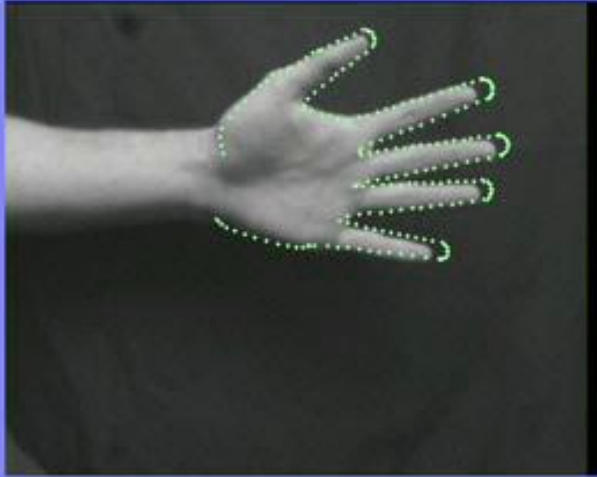


# Model-Based Hand Tracking Using an Unscented Kalman Filter

Björn Stenger and Roberto Cipolla

# The Problem

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# Contributions

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- What's new?
  - Regh & Kanade, ECCV'94
    - Truncated cylinders, no self-occlusion, 10 Hz.
  - Heap & Hogg, F&G'96
    - Point mesh, PCA, invalid motions, 10 Hz.
  - Isard & McCormic, ECCV'00
    - 2D b-spline, partitioned sampling, real-time.
  - Wu, Lin & Huang, ICCV'01
    - Data-glove + PCA, MC, view-dependent.
  - Stenger, Mendonça, Cipolla, CVPR'01
    - Accurate model, self-occlusion, UKF, 12 Hz.

# Contributions

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- Construction of hand model from truncated quadrics
- Contour generation handling self-occlusion
- Application of Unscented Kalman filter
- Tracking 7 DOF with 12Hz using 2 cameras

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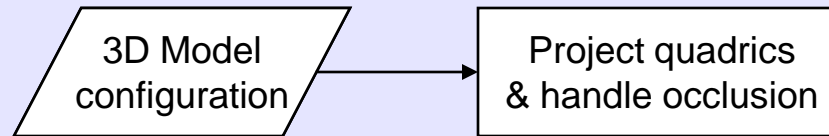
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# System Overview

---

Model  
Projection

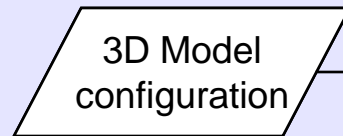




# System Overview

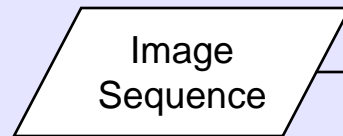
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Model  
Projection



Project quadrics  
& handle occlusion

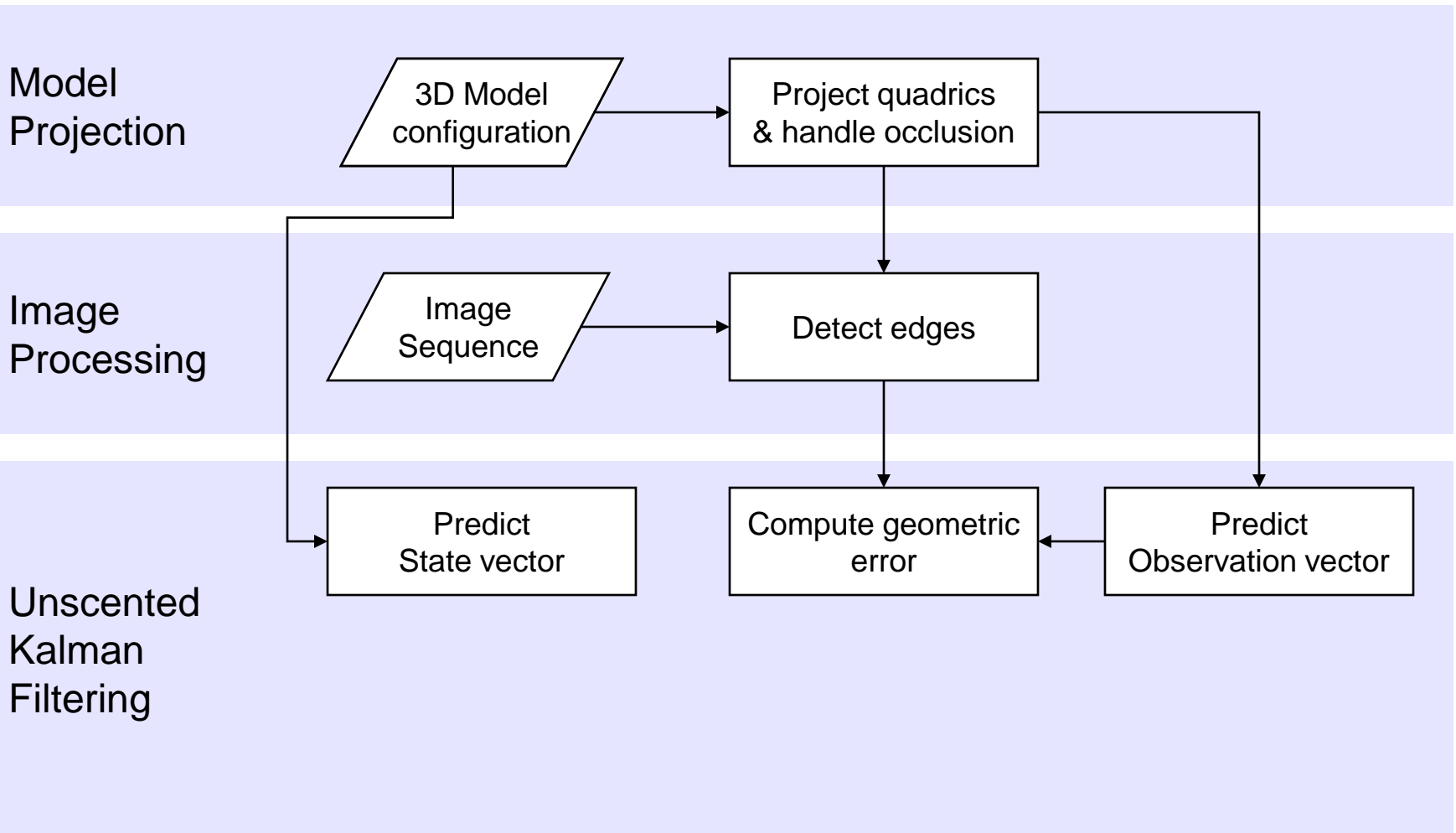
Image  
Processing



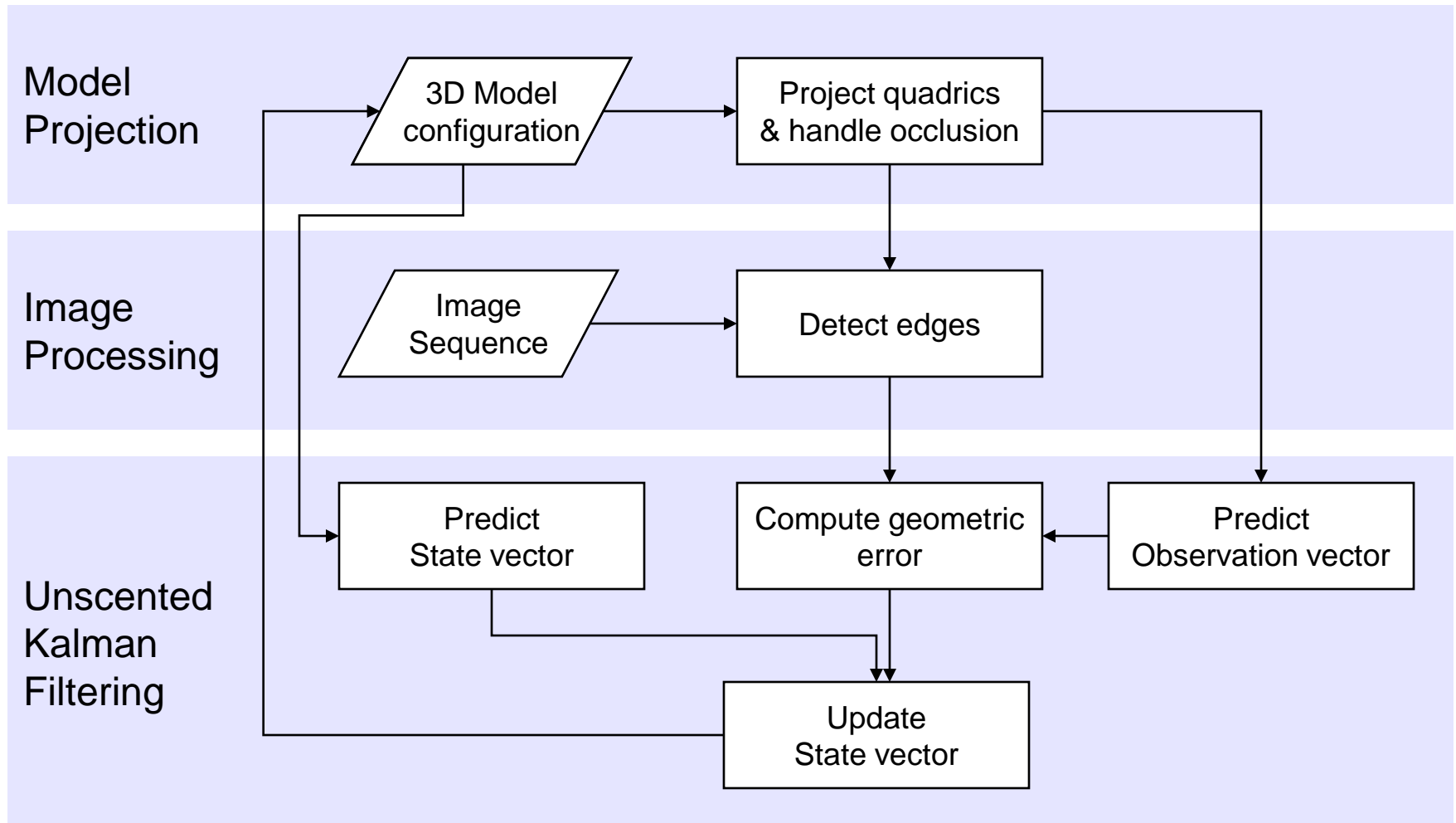
Detect edges



# System Overview



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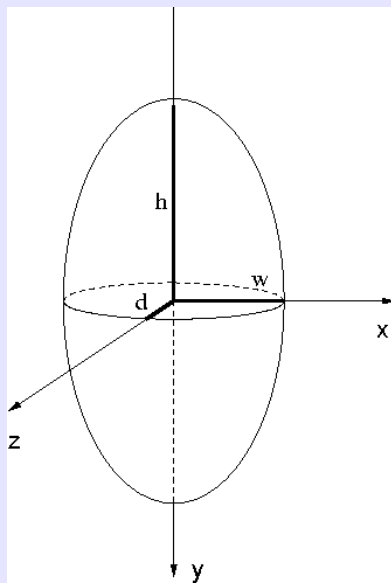


# Quadrics

**Quadric:** Second degree implicit surface defined by points  $\mathbf{X}$  satisfying  $\mathbf{X}^T \mathbf{Q} \mathbf{X} = 0$ .

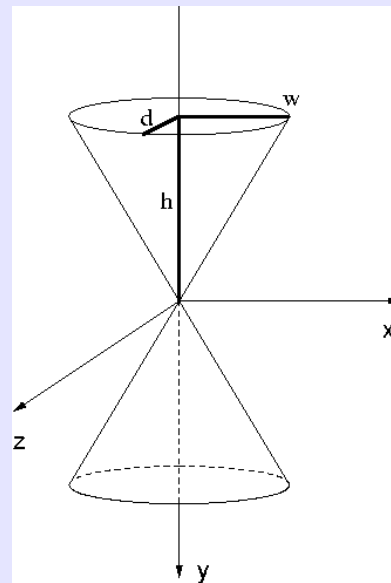
$$\text{rank}(\mathbf{Q}) = 4$$

Ellipsoid

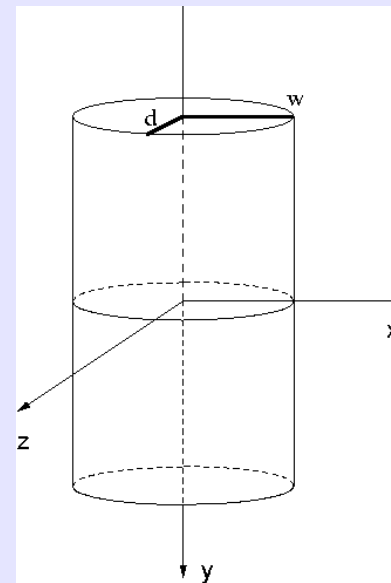


$$\text{rank}(\mathbf{Q}) = 3$$

Cone

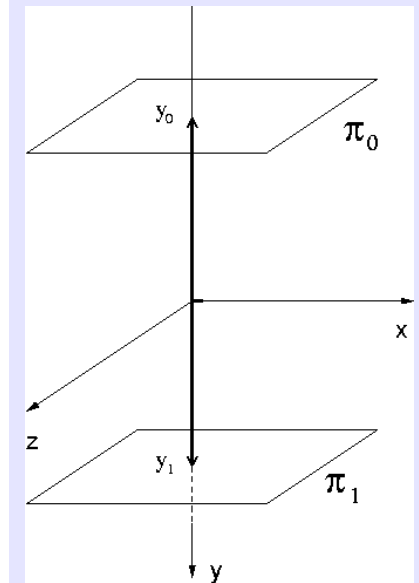


Cylinder



$$\text{rank}(\mathbf{Q}) = 2$$

Pair of Planes



# Shaping Quadrics

The shape of the quadric appear by factorizing  $\mathbf{Q}$ :

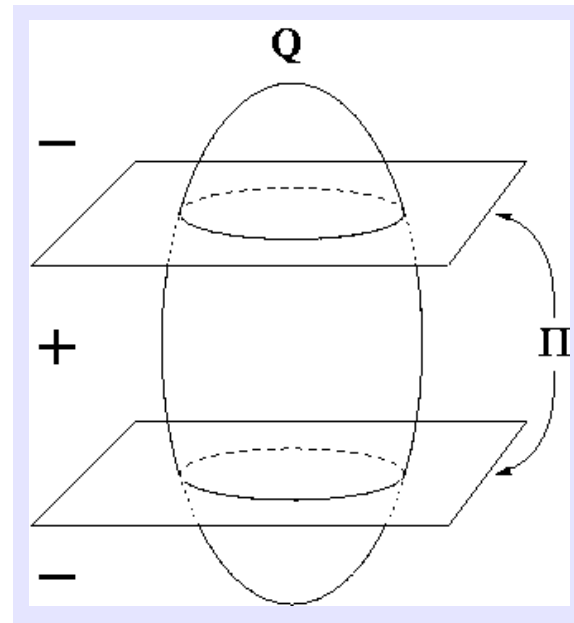
$$\mathbf{Q} = \underbrace{\begin{bmatrix} \mathbf{R} & \mathbf{0} \\ \mathbf{t}^T & 1 \end{bmatrix}}_{\text{Motion}} \underbrace{\begin{bmatrix} 1/a & 0 & 0 & 0 \\ 0 & 1/b & 0 & 0 \\ 0 & 0 & 1/c & 0 \\ 0 & 0 & 0 & 1/d \end{bmatrix}}_{\text{Shape}} \underbrace{\begin{bmatrix} \mathbf{R}^T & \mathbf{t} \\ \mathbf{0}^T & 1 \end{bmatrix}}_{\text{Motion}}$$

# Clipping Quadrics

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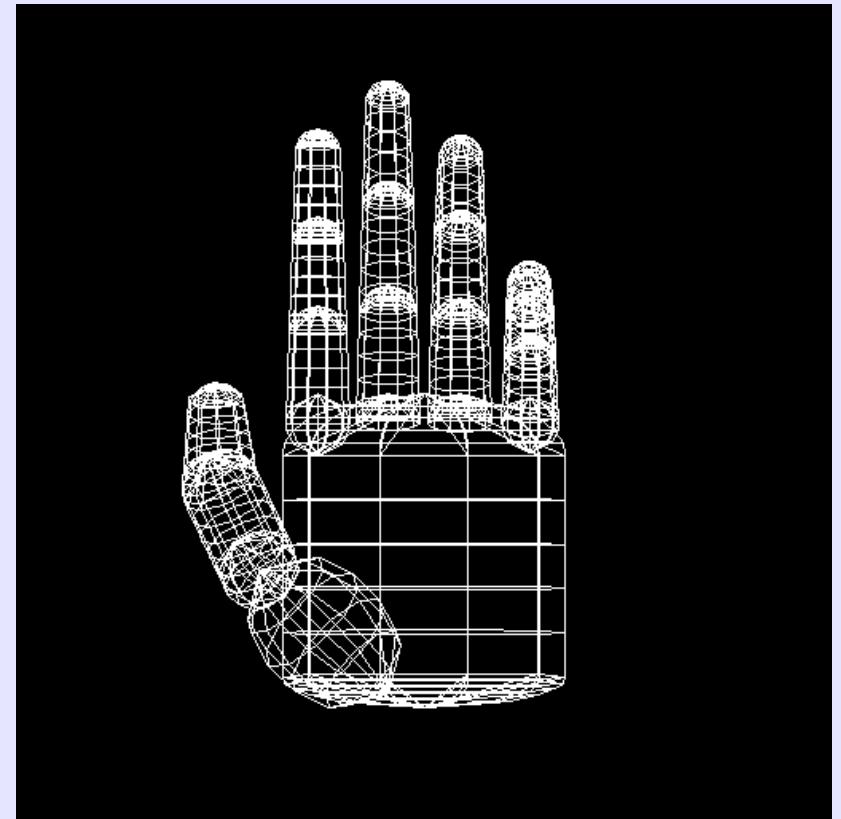
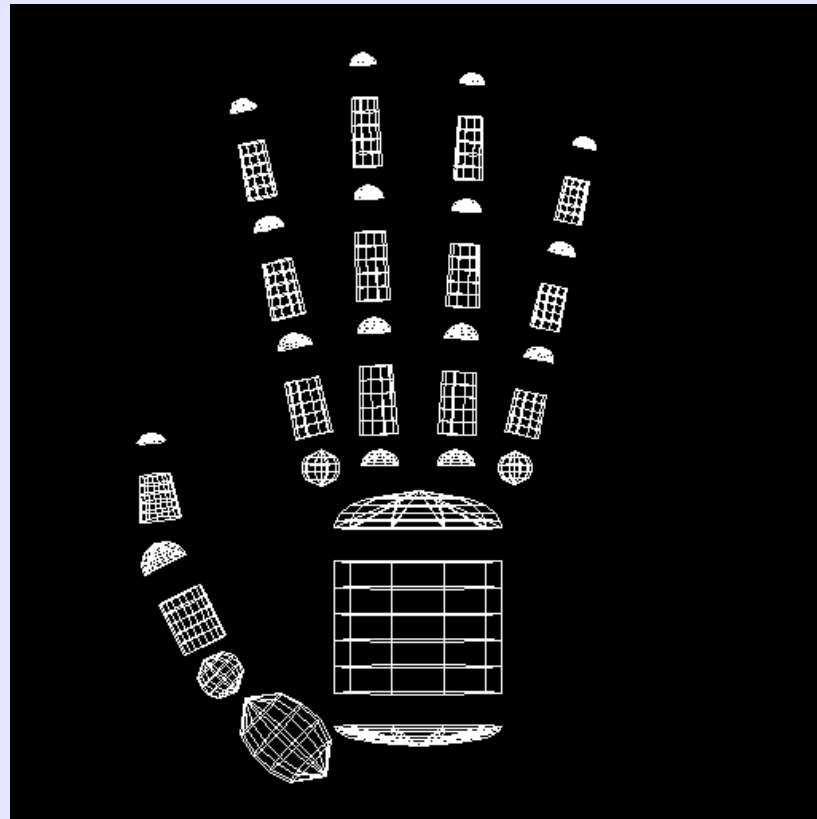
For modelling more general shapes  
truncate quadrics by finding points  $X$   
which satisfy:

$$\mathbf{X}^T \mathbf{Q} \mathbf{X} = 0 \quad \text{and} \quad \mathbf{X}^T \mathbf{\Pi} \mathbf{X} \geq 0$$



# Hand Model

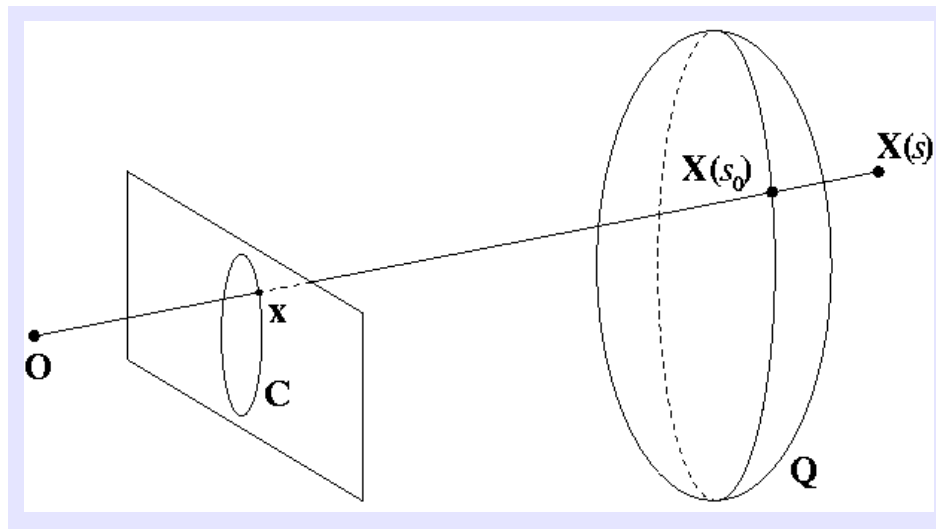
- 37 truncated quadrics
- 27 degrees of freedom (currently only 7 are tracked)



# Projection of a Quadric

Assuming a normalized projective camera  $\mathbf{P} = [\mathbf{I} \mid \mathbf{0}]$

Parameterize 3D points  $\mathbf{X}(s) = \begin{bmatrix} \mathbf{x} \\ s \end{bmatrix}$



$$\mathbf{X}^T(s) \mathbf{Q} \mathbf{X}(s) = 0$$



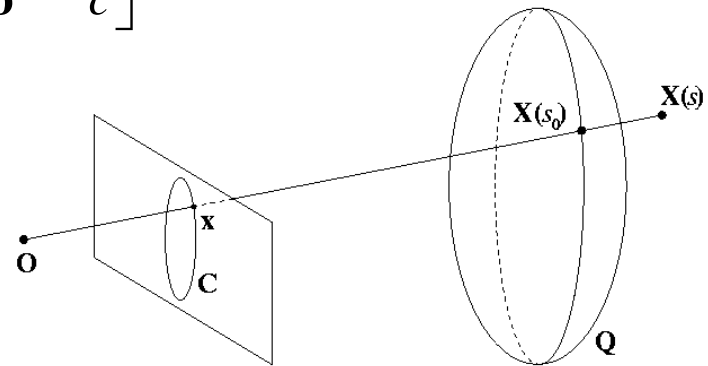
# Projection of a Quadric (2)

$$\mathbf{X}^T(s) \mathbf{Q} \mathbf{X}(s) = 0$$

$$\begin{bmatrix} \mathbf{x} & s \end{bmatrix} \begin{bmatrix} \mathbf{A} & \mathbf{b} \\ \mathbf{b}^T & c \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ s \end{bmatrix} = 0$$

$$cs^2 + 2\mathbf{b}^T \mathbf{x}s + \mathbf{x}^T \mathbf{A} \mathbf{x} = 0$$

$$\mathbf{Q} = \begin{bmatrix} \mathbf{A} & \mathbf{b} \\ \mathbf{b}^T & c \end{bmatrix}$$



Condition for  $\mathbf{X}(s)$  to be on the contour generator of  $\mathbf{Q}$ :

$$\Delta = 0 \Leftrightarrow \mathbf{x}^T (c\mathbf{A} - \mathbf{b}\mathbf{b}^T) \mathbf{x} = 0$$

$$\mathbf{x}^T \mathbf{C} \mathbf{x} = 0$$

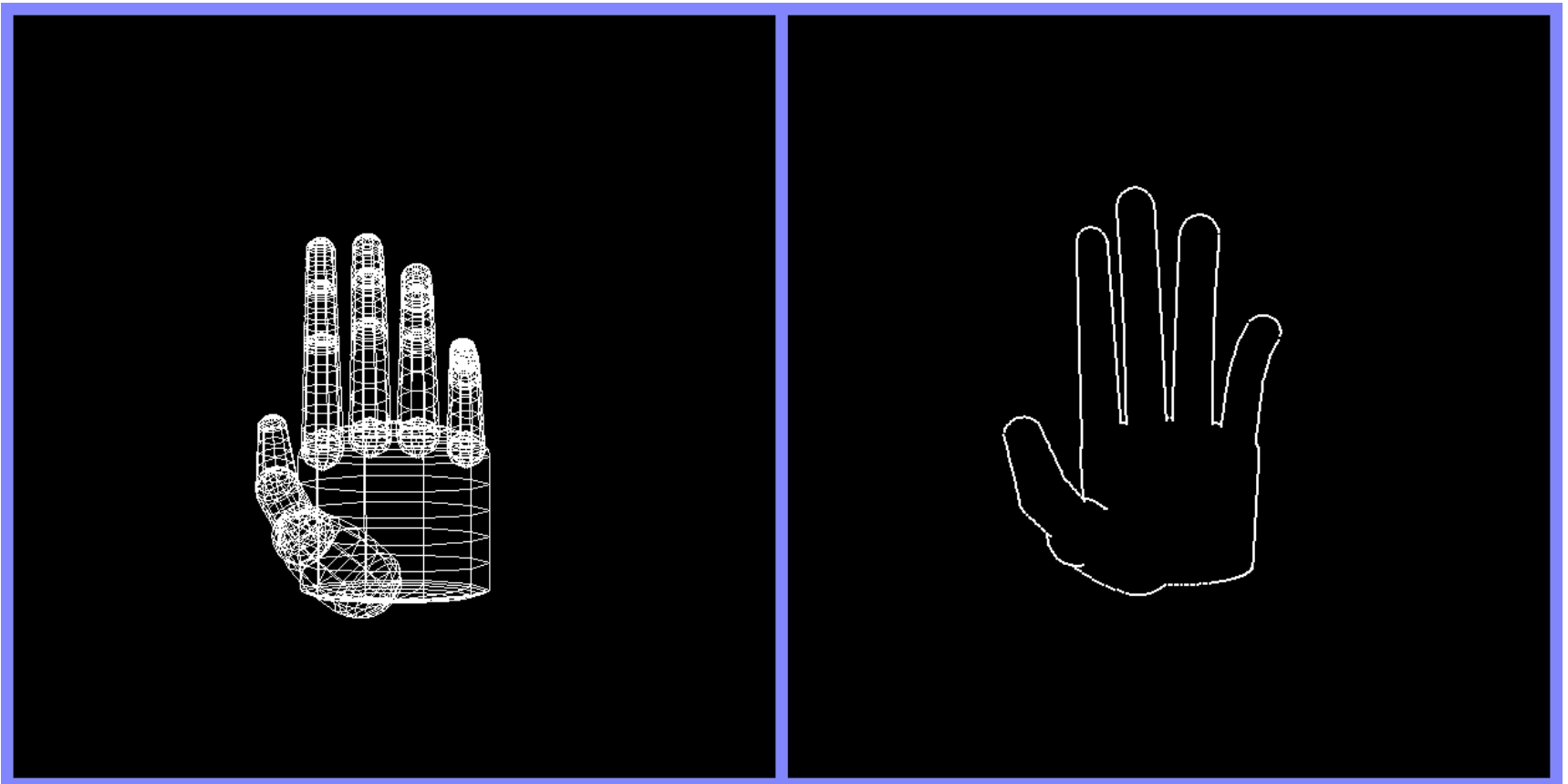
$$\mathbf{C} = c\mathbf{A} - \mathbf{b}\mathbf{b}^T$$

# Projecting the Hand Model

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3D model

Contours



# Optimal Filtering

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- Given a state-space model

$$\begin{aligned}\mathbf{x}_k &= \mathbf{f}(\mathbf{x}_{k-1}, \mathbf{v}_{k-1}) \\ \mathbf{z}_k &= \mathbf{h}(\mathbf{x}_k, \mathbf{w}_k)\end{aligned}$$

- Maximum likelihood estimator

$$\hat{\mathbf{x}}_k = \arg \min(-\log L(\mathbf{x}_k | \mathbf{z}_k, \dots, \mathbf{z}_0))$$

- For linear models, Gaussian error model: Kalman filter

$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k (\mathbf{z}_k - \hat{\mathbf{z}}_{k|k-1})$$

# The Unscented Transform

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Method for computing statistics of a random variable after a nonlinear transformation (Julier & Uhlmann, 1995).

Given:  $n$ -dimensional random variable  $\mathbf{x}_{k-1}$   
with mean  $\hat{\mathbf{x}}_{k-1}$  and covariance  $\mathbf{P}_{k-1}$

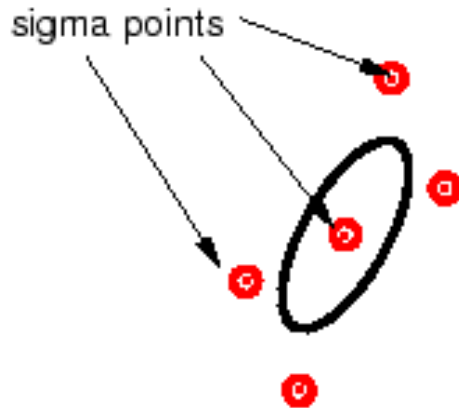
1. Compute  $2n+1$  points with associated weights.
2. Apply the nonlinear transformation to each point.
3. Compute mean and covariance of the transformed points.

# The Unscented Transform

## 1. Choose $2n+1$ points with associated weights

$$X_{k-1}^i = \begin{cases} \hat{\mathbf{x}}_{k-1} & i = 0 \\ \hat{\mathbf{x}}_{k-1} - \sigma_{k-1}^i & i = 1, \dots, n \\ \hat{\mathbf{x}}_{k-1} + \sigma_{k-1}^i & i = n+1, \dots, 2n \end{cases} \quad W_{k-1}^i = \begin{cases} 1/(n+1) & i = 0 \\ 1/2(n+1) & i = 1, \dots, 2n \end{cases}$$

where  $\sigma_{k-1}^i$  is the  $i^{\text{th}}$  column of the matrix  $\sqrt{(n+1)\mathbf{P}_{k-1}}$



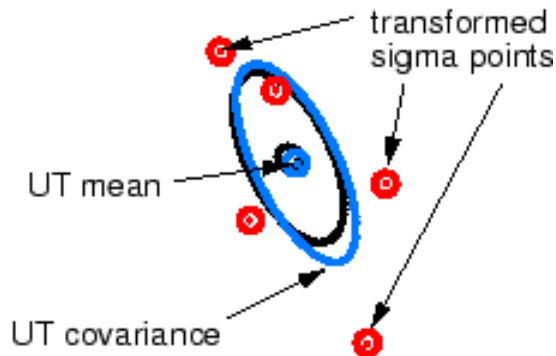
The set of points has the same mean, covariance and all higher odd central moments as the Gaussian distribution of  $\mathbf{x}_{k-1}$ .

# The Unscented Transform

2. Apply the nonlinear transformation to each point

$$X_{k|k-1}^i = f(X_{k-1}^i, k) \quad i = 0, \dots, 2n$$

3. Compute mean and covariance of the transformed points.



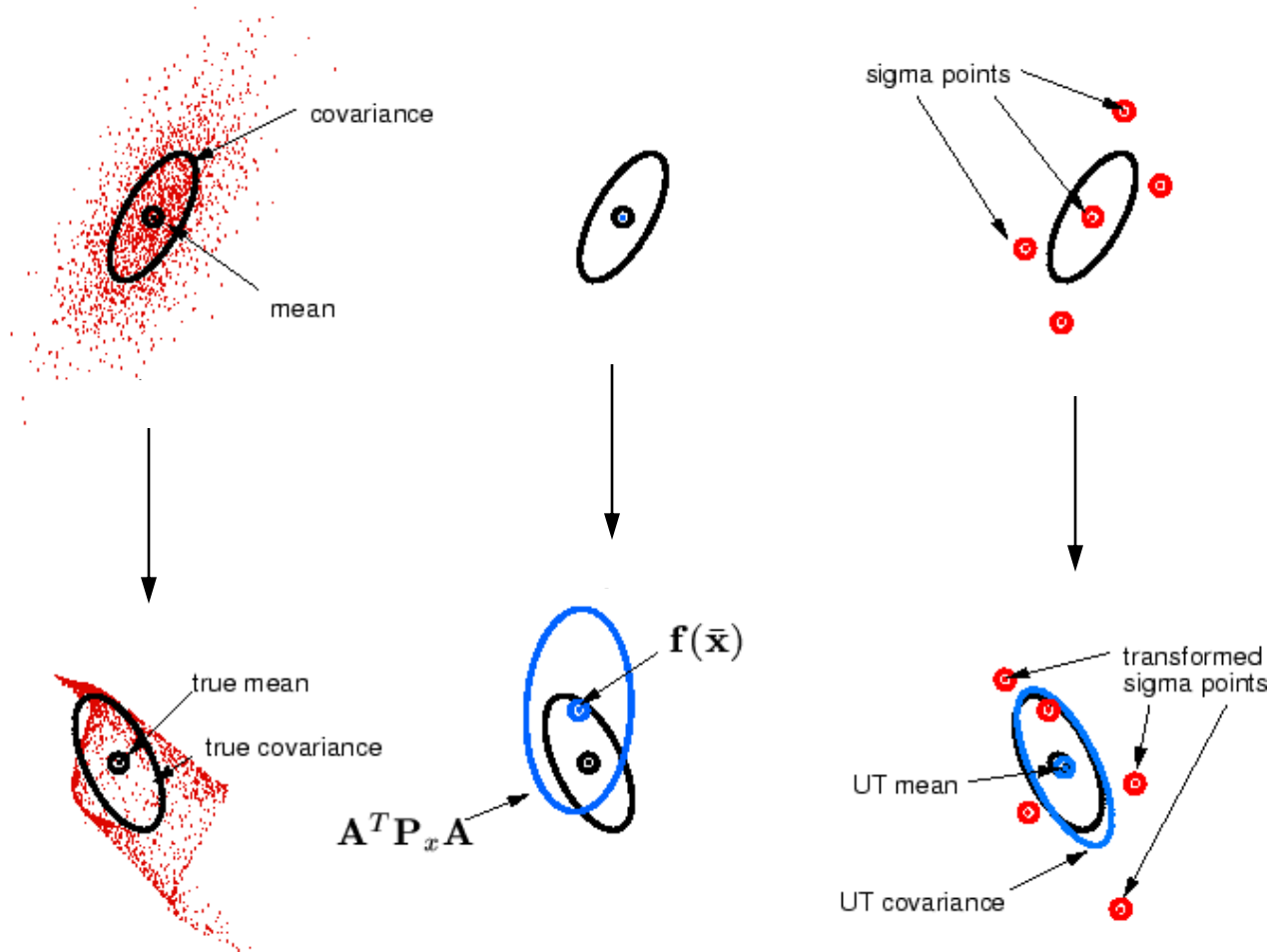
This approximation is correct up to the second order.

# Comparison

Actual (sampling)

Linearisation

Unscented Transform



# Properties of the UT

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- Approximates the distribution rather than the nonlinearity.
- Accurate to at least 2<sup>nd</sup> order (3<sup>rd</sup> order for Gaussian distributions).
- No Jacobian or Hessian matrices are needed.
- Efficient “sampling approach”.
- Assumes unimodal distributions.



# Unscented Kalman Filter

## Prediction Step:

- Prediction of state (and error covariance matrix)

$$\hat{\mathbf{x}}_{k-1} \xrightarrow{\text{Unscented Transform}} \hat{\mathbf{x}}_{k|k-1}$$

- Prediction of observation

$$\hat{\mathbf{z}}_{k-1} \xrightarrow{\text{Unscented Transform}} \hat{\mathbf{z}}_{k|k-1}$$

## Measurement Update Step:

- Compute innovation

$$\mathbf{v}_k = \mathbf{z}_k - \hat{\mathbf{z}}_{k|k-1}$$

- Compute Kalman gain  $\mathbf{K}_k$
- Update state estimation (and error covariance matrix)

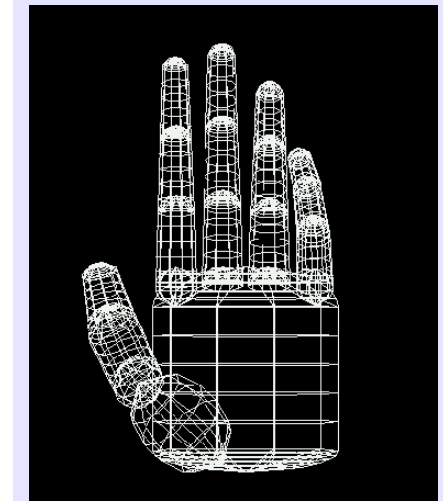
$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k \mathbf{v}_k$$

# State & Observation Vector

## State Vector $\mathbf{x}_k$ :

- Global pose parameters (6 DOF)
- Configuration of joints (1 DOF)
- Velocity and acceleration

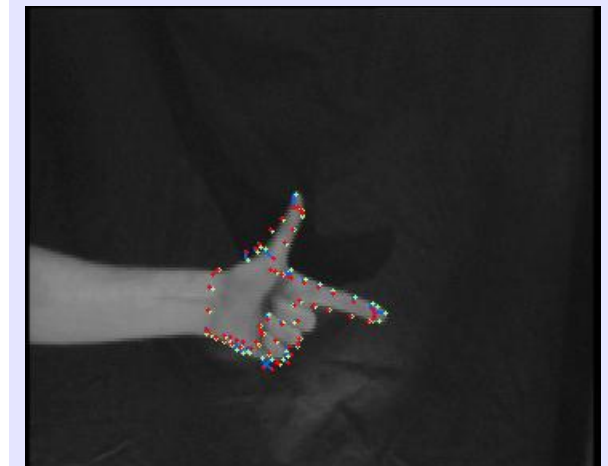
$$\mathbf{x}_k = [x_0, \dots, x_l, \dot{x}_0, \dots, \dot{x}_l, \ddot{x}_0, \dots, \ddot{x}_l]^T$$



## Observation Vector $\mathbf{z}_k$ :

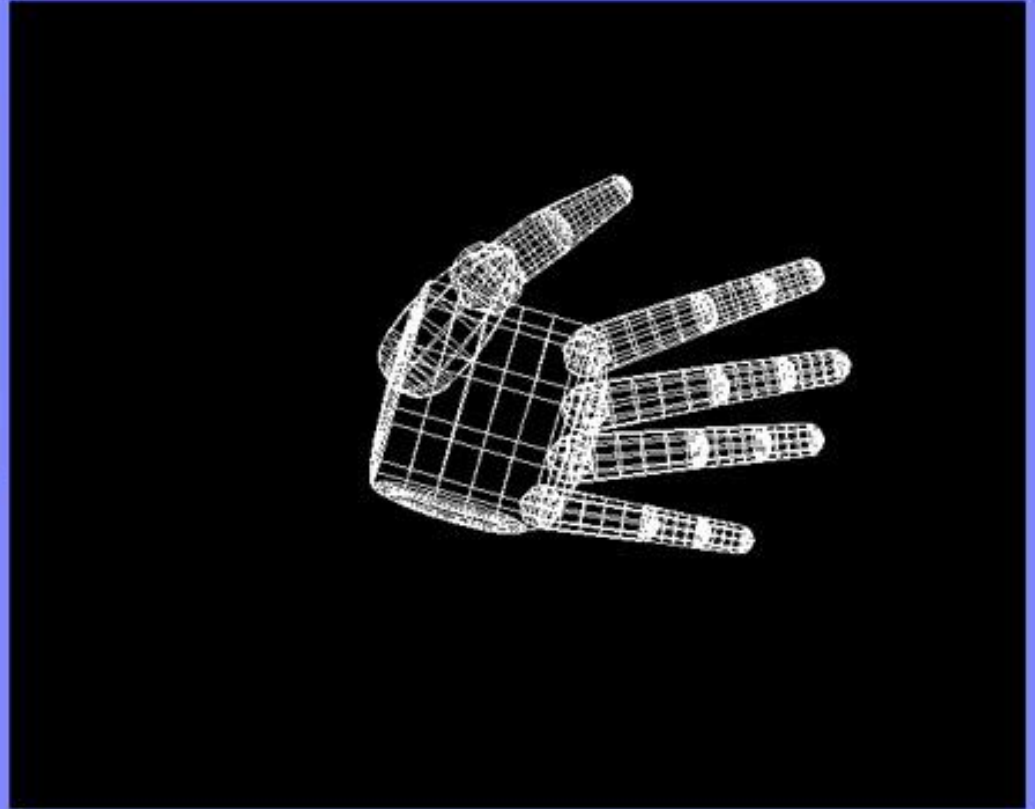
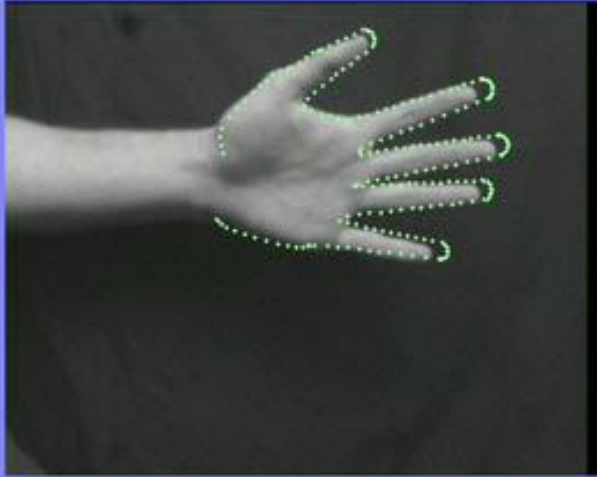
- Local edge detection at contour points

$$\mathbf{z}_k = \begin{bmatrix} \mathbf{n}_0^T \mathbf{s}_0 \\ \vdots \\ \mathbf{n}_m^T \mathbf{s}_m \end{bmatrix}$$



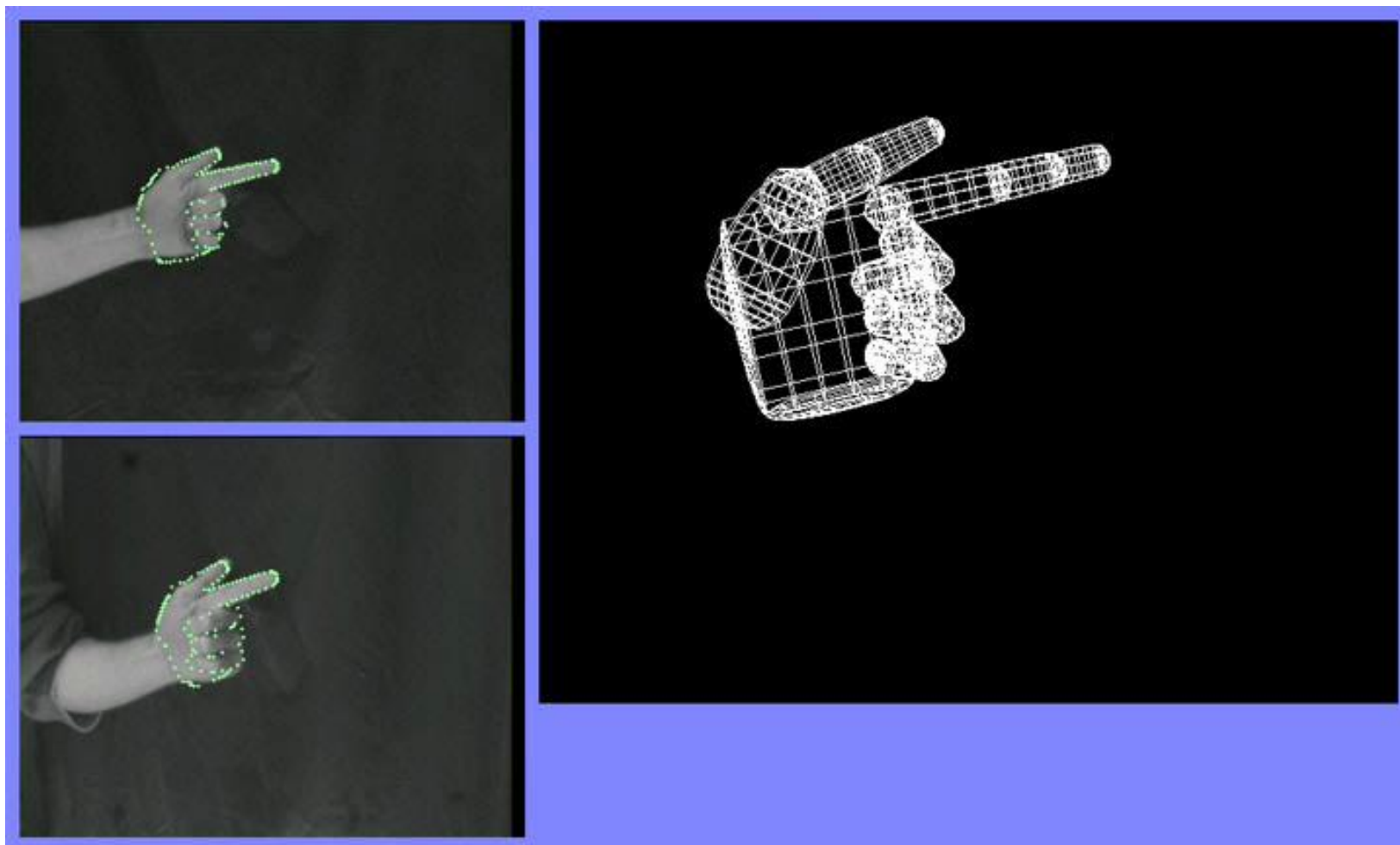
# Single View Tracking

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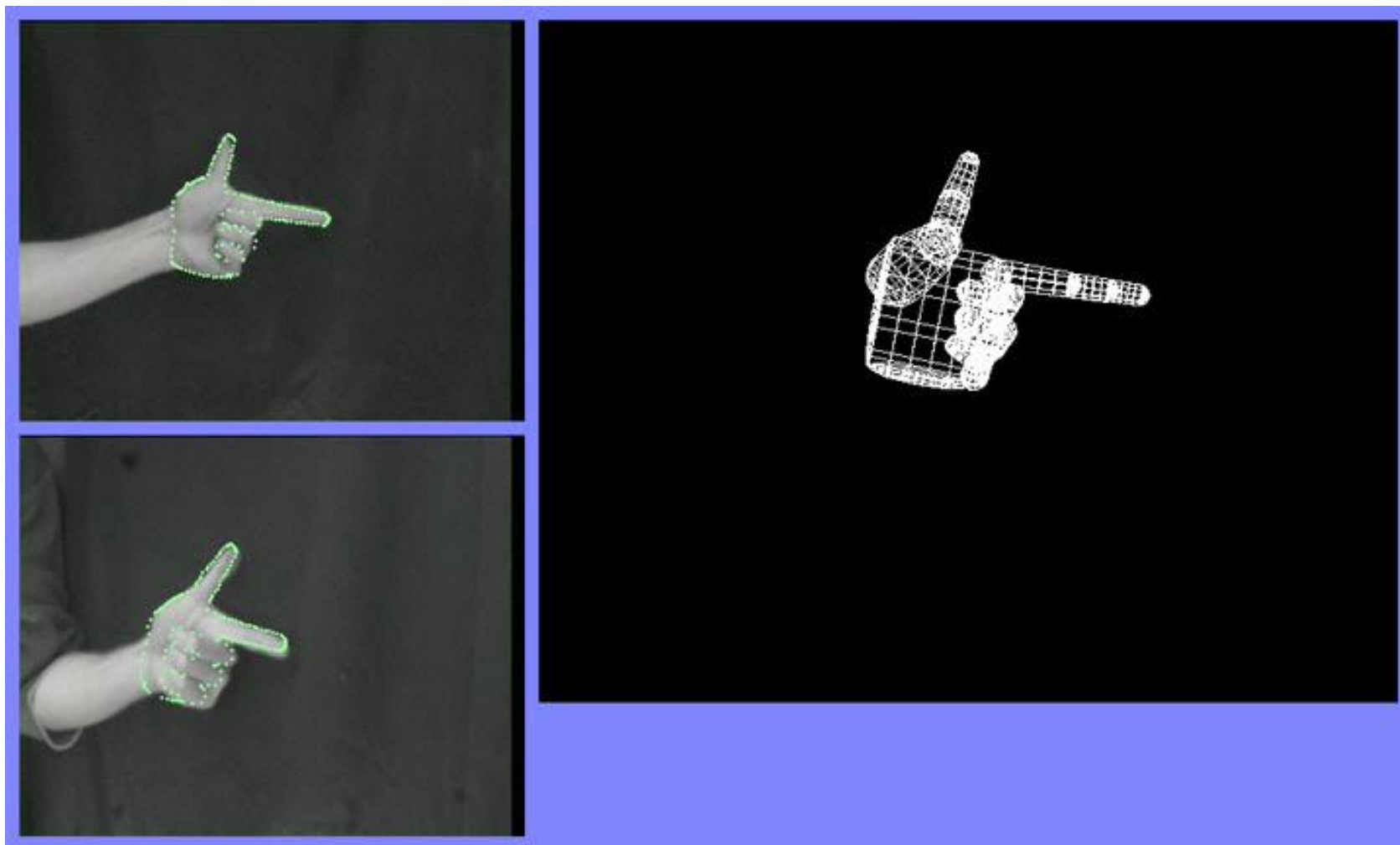
# Stereo Tracking 1

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# Stereo Tracking 2

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# Conclusions & Future Work

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- Construction of hand model from quadrics
- Handling self-occlusion
- Application of Unscented Kalman filtering
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- Increase DOFs
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- Handle cluttered backgrounds

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Thanks!

