

# Detecting Bipedal Motion from Correlated Probabilistic Trajectories

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## Abstract

This paper is about detecting bipedal motion in video sequences by using point trajectories in a framework of classification. Given a number of point trajectories, we find a subset of points which are arising from feet in bipedal motion by analysing their spatio-temporal correlation in a pairwise fashion. To this end, we introduce *probabilistic trajectories* as our new features which associate each point over a sufficiently long time period in the presence of noise. They are extracted from directed acyclic graphs whose edges represent temporal point correspondences and are weighted with their matching probability in terms of appearance and location. The benefit of the new representation is that it practically tolerates inherent ambiguity for example due to occlusions. We then learn the correlation between the motion of two feet using the probabilistic trajectories in a decision forest classifier. The effectiveness of the algorithm is demonstrated in experiments on image sequences captured with a static camera, and extensions to deal with a moving camera are discussed.

*Keywords:* motion, trajectories, spatio-temporal features, decision forest

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## 1. Introduction

1 Point motion in an image sequence not only gives strong cues about the  
2 underlying geometry in 3D space, but may also be characteristic for an ob-  
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4 ject class to which the point belong. Further, given a multiple of 2D point  
 5 patterns, we can infer far more information than we would obtain from a  
 6 single point trajectory through the correlation between the motion patterns.  
 7 Hence, trajectories of points in image sequences provide a strong visual cue,  
 8 often allowing the human brain to infer the scene behind those points. For  
 9 example, when points close to the joints of a walking person are tracked,  
 10 the psychological effect of *kinetic depth* allows us to perceive walking motion  
 11 solely from the 2D point motion pattern. This has first been studied by Jo-  
 12 hansson using moving light displays (MLDs) (Johansson, 1973). The goal in  
 13 this paper is to achieve this recognition ability for detecting pedestrian mo-  
 14 tion from tracked points on a pair of feet whose trajectories are characteristic  
 15 and spatio-temporally correlated.

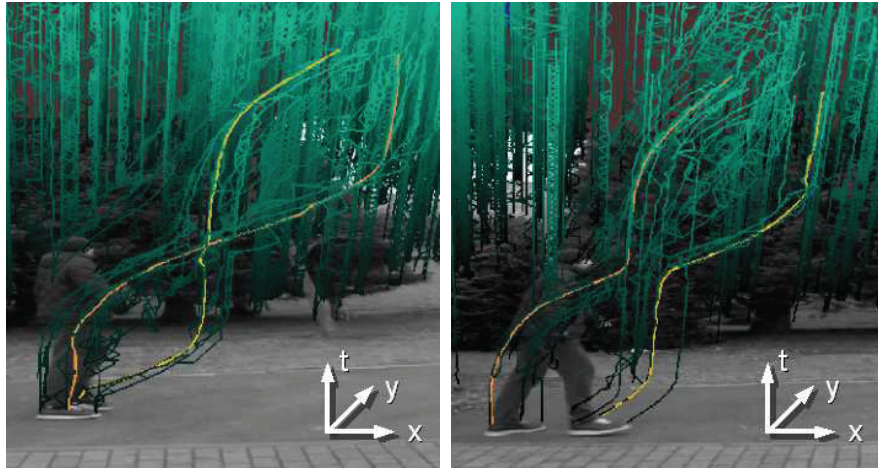


Figure 1: **Space-time volume with point trajectories.** See the supplementary video for further explanations of space-time representation. Brighter green indicates corner positions more recent in time. Left: Two sample trajectories of corners on the feet are highlighted in yellow. Right: Another case where features are swapped during the short occlusion. Our method is able to correctly classify both cases as walking motion.

16 The work presented in this paper can be categorized as motion-based  
 17 recognition, but is unique in the sense that we do not assume clean point  
 18 tracks. In order to obtain a discriminative trajectory, in this application, a  
 19 point should ideally be tracked during a complete walk cycle (about one sec-  
 20 ond). However, point trajectories of typical outdoor scenes are rarely reliable

21 over such a long period of time. Therefore, we retain the concept of tem-  
22 poral connectedness by introducing the notion of *probabilistic trajectories*.  
23 See Figure 1 for an example of such trajectories. These are sampled from  
24 a directed acyclic graph whose edges represent temporal point correspon-  
25 dences weighted by their matching probability in terms of appearance and  
26 location. We choose to use standard corner features (Harris and Stephens,  
27 1988) for the points to track rather than space-time interest points (Laptev  
28 and Lindeberg, 2003) in order to continue detecting points even when they  
29 are stationary during the walk cycle. Temporal correspondence is thus hy-  
30 pothesized while sacrificing matching accuracy in order to obtain longer and  
31 more discriminative trajectories. The advantage of this representation is that  
32 it permits inherent trajectory ambiguity, for example due to occlusions, and  
33 practically gain richer representations of the scene which facilitate the tasks  
34 of recognition using motion.

35 Physical models of bipedal motion have recently been used in tracking  
36 a walking person (Brubaker et al., 2010). Intuitively, the motion of a point  
37 on a single foot is composed of two periods of dynamic and static phases  
38 (Bissacco, 2005) and motion of points from a pair of feet are alternating in  
39 a cyclic manner. We aim to directly learn to detect this type of foot motion  
40 in a discriminative manner. The key idea for our bipedal motion detection  
41 from trajectories is thus to detect *correlated spatio-temporal features*. That  
42 is, we detect pedestrian motion by observing the correlation between the  
43 motion of two feet of the same person. It is in contrast to (Brostow and  
44 Cipolla, 2006) whose premise is that a pair of points that appear to move  
45 together are part of the same individual. The motivation behind our strategy  
46 is the fact that bipedal motion is essential to any walking person in terms  
47 of physical dynamics of walking motion whereas the motion of other body  
48 parts such as the arms typically exhibits significantly more variation. To the  
49 best of our knowledge this is the first attempt<sup>1</sup> to recognize motion by way  
50 of investigating correlation of point trajectories.

51 In this work we opt for a learning based approach and develop a classifier  
52 for bipedal motion of a pair of feet among a number of point trajectories.  
53 We employ a decision forest classifier (Breiman, 2001; Geurts et al., 2006;  
54 Ho, 1998) which has been successfully applied to different classification tasks  
55 (Brostow et al., 2008; Lepetit et al., 2005; Rogez et al., 2008; Shotton et al.,

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<sup>1</sup>An early description of this work has appeared in (Perbet et al., 2009).

2008). The reason of using it is in the ease of training, the ability to deal with a large amount of data, and good generalisation performance. It is also well suited to our probabilistic input. That is, we use sampled subgraphs of probabilistic trajectories as input data. We build a two-stage decision forest classifier. The first stage identifies candidate foot trajectories and the second stage associates candidate trajectories as pairs, effectively exploiting the correlated spatio-temporal features.

### 1.1. Related Work

A lot of work in the area of motion-based recognition including human gait analysis have been inspired by the biological phenomenon (Cédras and Shah, 1995; Gavrilu, 1999). These methods typically require a robust method for feature extraction. Thus, trajectories of interest points were either obtained by using markers to simplify image analysis (Campbell and Bobick, 1995) or acquired from motion captured data (Meng et al., 2006) in place of MLDs; automatic acquisition of accurate point tracks is difficult in many cases due to effects such as occlusions, lighting changes and image noise (BenAbdelkader et al., 2002). Although relatively little work on recognition were performed purely from low-level features extracted from natural image sequences (Polana and Nelson, 1994), increasing challenges in computing and applying point trajectories have been recently presented (Sand and Teller, 2008; Perbet et al., 2009; Messing et al., 2009; Matikainen et al., 2009; Sun et al., 2010; Sundaram et al., 2010; Wu et al., 2011; Wang et al., 2011) especially in the context of action recognition.

Two of the common critical factors for computing reliable trajectories are, however, to select good repeatable features that are relevant to motion recognition, and to maintain them continuous throughout a required part of the given image sequence. In this respect, Kanade-Lucas-Tomasi (KLT) tracker (Shi and Tomasi, 1994) is a popular option and utilized in (Messing et al., 2009; Matikainen et al., 2009; Sun et al., 2010) although obtained trajectories inevitably suffer from discontinuous to an extent according to the noise and clutters. One solution to deal with discontinuities could be to generate shorter but reliable ‘tracklets’ (Ge and Collins, 2008) and to link them in an additional step (Huang et al., 2008). Other extentions have been for computing trajectories in a dense manner, typically with incorporation of optical flow; particle video (Sand and Teller, 2008) is one of the early such representations. For the same goal, the work in (Sun et al., 2010) combines KLT with SIFT-trajectories (Sun et al., 2009), and particle trajectories (Wu

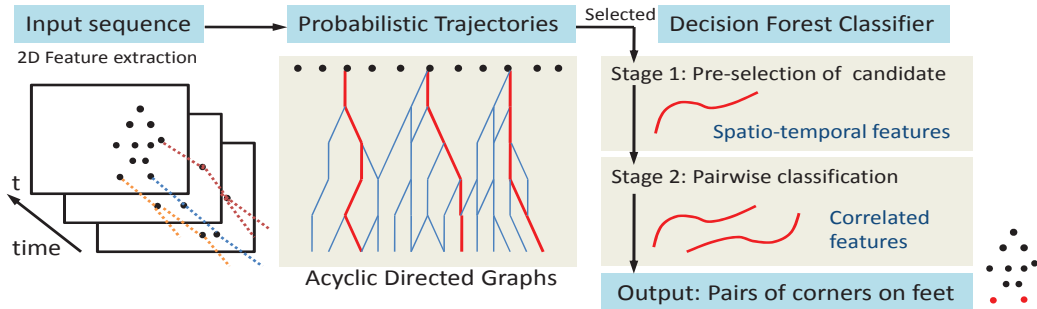


Figure 2: **Schematic of the algorithm.** Given a video sequence and 2D corners detected in each frame, we first sample probabilistic trajectories of corners in the graph, and then classify the trajectories by a two-stage decision forest. We design correlated spatio-temporal features for classification.

93 et al., 2010) have been used in (Wu et al., 2011). Also, a real-time dense  
 94 point tracker (Sundaram et al., 2010) has been developed based on on large  
 95 displacement optical flow (Brox and Malik, 2011). As mentioned earlier, nev-  
 96 ertheless, there are intrinsic difficulties in maintaining correct and consistent  
 97 point correspondence in generating long trajectories, most notably due to  
 98 occlusions. In order to have a continuum of their 2D coordinate in the form  
 99 of a trajectory, it will be indispensable to somehow reinforce correspondence.  
 100 This is especially important when the correlation need be analysed between  
 101 trajectories of points which can often occlude with each other. The proba-  
 102 bilistic trajectories introduced in this paper are designed by prioritising the  
 103 concept of temporal connectedness, and to utilize what we perceive from 2D  
 104 motion of points in natural images for recognition.

### 105 1.2. Contributions and Assumptions

106 Figure 2 shows a schematic of our algorithm. The contributions of the  
 107 paper are four-fold: (i) the introduction of *probabilistic trajectories* which  
 108 temporally associate each point over a sufficiently long time period under  
 109 both image noise and occlusion, (ii) the pairwise analysis of trajectories for  
 110 detecting characteristic correlation between the two feet in bipedal motion,  
 111 (iii) the design of efficient features which are computed in the two-staged  
 112 decision forest classifier, and (iv) a discussion on potential extensions to deal  
 113 with camera motion.

114 We do not assume clean point tracks but instead assume that the camera  
 115 captures dynamics of motion at sufficiently high rate (we use 60 fps) and  
 116 that people walk with approximately constant speed and direction during a  
 117 gait cycle.

## 118 2. Probabilistic Trajectories

119 In this section, we describe the three successive steps that generate *prob-*  
 120 *abilistic trajectory*. The basic idea is to hypothesize trajectories by enforcing  
 121 temporal correspondences between consecutive frames since repeated detec-  
 122 tion of the same point over a long time interval is not always possible. In  
 123 practice, we generate an acyclic graph for each point of interest using it as  
 124 the root node and grow a graph such that each edge represents a possible  
 125 temporal correspondence. Corners of two consecutive frames are connected  
 126 probabilistically using their spatial distance and their appearance. Over  $T$   
 127 frames, those connections form a graph of possible trajectories. A walk in  
 128 this graph describes a possible trajectory of a given corner over time, also  
 129 including many incorrect trajectories. Our assumption is that most of these  
 130 will still be discriminative, see Figure 1, right. Different paths from the root  
 131 node toward the leaf nodes give different trajectories, allowing for an inherent  
 132 ambiguity in the matching process. For example, although some trajectories  
 133 may reflect apparent motion rather than real motion, this is encoded in a  
 134 probabilistic manner. This matching process ensures that at each time step  
 135 trajectories of equal length are available for all points in the frame.

### 136 2.1. Matching Between Two Consecutive Frames

137 In every frame we extract Harris corners (Harris and Stephens, 1988)  
 138 and find potential ancestors for each point among the feature set from the  
 139 previous frame. Let  $p_i(t), i = 1, \dots, n$  be the  $i^{\text{th}}$  corner detected at a 2D  
 140 location  $\mathbf{x}_i(t) \in \mathbf{R}^2$  at time  $t$  and let  $p_j(t-1), j = 1, \dots, m$  be the  $j^{\text{th}}$  corner  
 141 found at  $\mathbf{x}_j(t-1)$  among  $m$  corners which were within a certain range from  
 142  $\mathbf{x}_i(t)$  in frame  $t-1$ . We then define the temporal matching score,  $P_{ij}(t)$ , that  
 143  $p_i(t)$  matches  $p_j(t-1)$  in terms of their appearance similarity  $S_{ij}$ , and the  
 144 spatial distance  $D_{ij}$ , by

$$P_{ij}(p_i(t), p_j(t-1)) \propto \exp(-\alpha S_{ij}) \exp(-\beta D_{ij}) , \quad (1)$$

145 where  $\alpha$  and  $\beta$  are positive weighting coefficients.

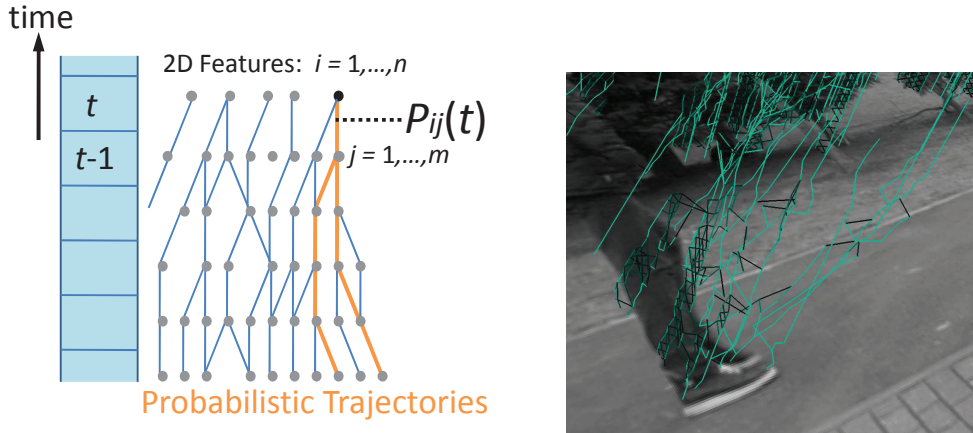


Figure 3: **Probabilistic Trajectories.** Left: A sketch of a graph and selected probabilistic trajectories as our motion descriptor. Right: An example of partial graph with varying color representing different probabilities, i.e. brighter indicates higher values.

146 The appearance similarity  $S_{ij}$  is computed from the local image regions  
 147 around  $p_i(t)$  and  $p_j(t-1)$ , respectively, as the SAD score between them (after  
 148 subtracting the mean intensity of each image patch) and  $D_{ij}$  by their spatial  
 149 distance  $D_{ij} = \|\mathbf{x}_i(t) - \mathbf{x}_j(t-1)\|$ . The assumption that the camera captures  
 150 dynamics of motion at sufficiently high rate, 60 fps rather than usual 30  
 151 fps, is to ensure that for each corner point detected in frame  $t$  we can find  
 152 the corresponding corner in frame  $t-1$  within a reasonable range so that  
 153 relatively smooth probabilistic trajectories can be generated.

154 We represent the existence of a potential match between  $p_i(t)$  and  $p_j(t-1)$   
 155 as a binary value,  $E_{ij}(t) \in \{0, 1\}$ , based on  $P_{ij}(t)$  and define the match as  
 156 active,  $E_{ij}(t) = 1$ , with the condition:

$$P_{ij} > \max_j P_{ij} - e, \quad (2)$$

157 where the threshold value  $e$  is dynamically adjusted so that the number of  
 158 pairs is constant which is set to  $4n$ . Note that this may result in no active  
 159 matches for some corners with low values of  $\max_j P_{ij}$ . We also add temporal  
 160 matches for the same set of consecutive frames in the forward direction by  
 161 repeating the process in a reverse manner so that more potential matches are  
 162 ensured.

163 *2.2. Acyclic Graph with Matching Probabilities*

164 For each time step  $t$  we have determined temporal matches  $E_{ij}(t)$  between  
 165 corners across previous adjacent frames. We retain these for the last  $T$  frames  
 166 (the choice of  $T$  will be discussed later). Defining each point  $p_i(t)$  as a root  
 167 node, we generate an acyclic graph,  $\mathcal{G}_i(N, E)$ , of depth  $T$  by tracing active  
 168 temporal matches along the time axis backward for  $T$  frames, see Figure 3.  
 169 The graph  $\mathcal{G}_i(N, E)$  consists of nodes,  $N$ , which represent matched corners  
 170 in the preceding  $T$  frames, and edges,  $E$ , connecting these nodes. Namely,  
 171  $E = (E_{ij}(\tau), \tau = t, \dots, t - T + 1)$ . An edge representing an active match,  
 172  $E_{ij}(t)$ , has  $P_{ij}(t)$  as its associated weight.

173 Note that the number of frames,  $d$ , for which each corner can be traced  
 174 back (until encountering an inactive edge or a ‘dead end’) is available for each  
 175 node. For example,  $d[E_{ij}(t)] = 1$  if  $p_j(t-1)$  has no ancestor and  $E_{jk}(t-1) = 0$   
 176 for all  $k$  (where  $k$  is an index to features in frame  $t - 2$ ). We assign  $d$  to  
 177 each node as its attribute while ideally  $d > T$  where at least one path can  
 178 be found containing nodes from the  $T$  previous frames.

179 *2.3. Sampling Probabilistic Trajectories*

180 In each time step the graph is updated and trajectories are sampled from  
 181 it that are then classified. Intuitively, the sampled trajectories need to be  
 182 long and physically plausible. Now, we define the *probabilistic trajectories*,  
 183  $X_i(t) \in \mathbf{R}^{2T}$  of  $p_i(t)$ , as the paths connecting the root node to different  
 184 leaf nodes of  $\mathcal{G}_i(N, E)$ . In practice, a graph traversal of  $\mathcal{G}_i$  guided by a  
 185 probabilistic selection of edges at each node results in plausible trajectories.  
 186 In particular, we use the sampling probability,  $\hat{P}_{ij}$ , in which we also take into  
 187 consideration the traceable depth  $d$  and the velocity conservation factor,  $V_{ij}$ :

$$\hat{P}_{ij}(p_i(t), p_j(t-1)) \propto P_{ij} \exp\left(-\frac{\gamma}{d[E_{ij}]+1}\right) \exp(-\delta V_{ij}) , \quad (3)$$

188 where  $\gamma$  and  $\delta$  are positive weighting coefficients, and the last factor

$$V_{ij}(\tau) = \|(\mathbf{x}_h(\tau+1) - \mathbf{x}_i(\tau)) - (\mathbf{x}_i(\tau) - \mathbf{x}_j(\tau-1))\| \quad (4)$$

189 is valid when  $\tau < t$  (so that the coordinate of the previous node in the path,  
 190  $\mathbf{x}_h(\tau+1)$ , is available). We set  $\gamma = 10$  and  $\delta = 1$  in our experiments.

191 A sophisticated selection of paths such as in (Torresani et al., 2008) would  
 192 help if spatial coherence of matched points could be taken into account. In  
 193 our case, however, points on different feet have diverse spatial path and we  
 194 remain to find the path individually.



195 *2.4. Computational complexity*

196 We detect  $n$  corner points in each frame and use  $m$  corners in the previous  
197 frame for computing the correlation; the complexity of matching between two  
198 consecutive frames is  $O(nm)$ . However,  $m$  is set to be significantly smaller  
199 than  $n$ ; in our experiments we set  $n = 300$  and  $m = 10$ . Also, the total  
200 number of pairs that are considered to be parts of trajectories is adjusted to  
201  $4n$ . The complexity for the computation between consecutive frame can be  
202 seen as  $O(n)$  given  $n \gg m$ . For generating an acyclic graph and thereby  
203 sampling trajectories, we need to compute the velocity conservation factor  
204 as well as the traceable depth for the length of the trajectories  $T$ ; the com-  
205 plexity in this respect is  $O(T)$ , making the total computational complexity  
206 for computing the probabilistic trajectories  $O(nT)$ .

207 **3. Classification of Trajectories**

208 Given a corner,  $p_i(t)$ , and its probabilistic trajectory,  $X_i(t)$ , our task is  
209 now to determine whether or not  $X_i(t)$  is the trajectory of a foot during  
210 walking motion. In order for a trajectory to contain discriminative features,  
211 we consider its length  $T$  as approximately covering one walk cycle. As men-  
212 tioned above, the key idea is to observe point trajectories in pairs. That  
213 is, we also consider  $p_u(t) (u \neq i)$  that are located in the neighborhood of  
214  $p_i(t)$  and examine the spatio-temporal correlation between the probabilistic  
215 trajectories,  $X_i(t)$  and  $X_u(t)$ .

216 In order to avoid examining the large number of possible pairs we also  
217 use the fact that some trajectories can be rejected immediately as candi-  
218 dates, such as those from stationary background points or those that are too  
219 noisy due to incorrect temporal association. Thus, we employ a two-stage  
220 classification process:

- 221 1° Selection of candidate trajectories.
- 222 2° Pairwise classification of pertinent trajectories.

223 It should be noted that we benefit from the selection of the candidates in  
224 reducing the complexity in terms of the number of possible pairs to be con-  
225 sidered in the second stage, and therefore overall computational cost. In  
226 each stage, we need a classification tool and invariant features that allow  
227 us to distinguish trajectories of walking motion from others. We perform  
228 classification using decision forests in both stages. Decision forests are well

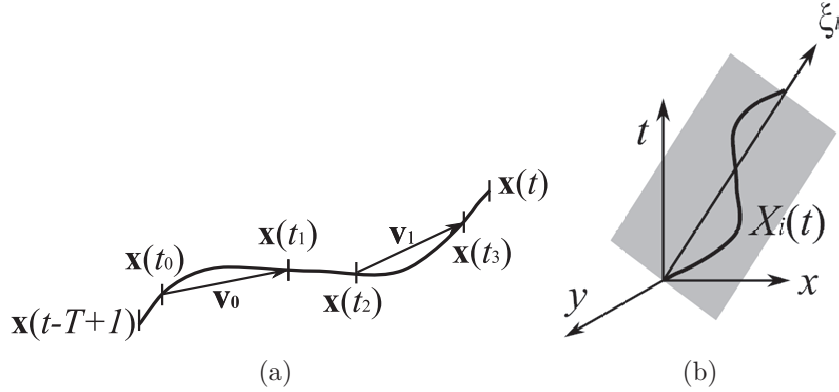


Figure 4: **Feature Vectors from Trajectories.** (a) In order to compute features we sample many pairs of velocity vectors from a trajectory. (b) The principal direction  $\xi$  of the trajectory  $X_i(t)$  is used for the directional feature computation.

229 suited to our task because each of our input instances is probabilistic, i.e.  
 230 any trajectories sampled as subgraphs are probabilistic in both stages, 1<sup>o</sup>  
 231 and 2<sup>o</sup>.

### 232 3.1. Selection of Candidate Trajectories

233 The goal in the first stage is to select candidate trajectories prior to pair-  
 234 wise classification. We thus carry out the selection of candidate trajectories  
 235 by individual  $X_i(t)$ . The feature design for this stage is based on the obser-  
 236 vation that  $X_i(t)$  originating from a foot is characterized by dynamic and  
 237 static phases, being distinguishable from simple trajectories coming from  
 238 background.

#### 239 3.1.1. Feature Vectors

240 Let a trajectory,  $X_i(t)$ , be represented by a vector  $X_i(t) = [\mathbf{x}(t), \mathbf{x}(t -$   
 241  $1), \dots, \mathbf{x}(t - T + 1)]^\top$ . We first remove its linear component,  $\bar{X}_i(t)$ , and convert  
 242  $X_i(t)$  to its canonical form,  $\tilde{X}_i(t) = [\tilde{\mathbf{x}}(t), \tilde{\mathbf{x}}(t - 1), \dots, \tilde{\mathbf{x}}(t - T + 1)]^\top$  (see  
 243 Appendix). The merit of using the canonical form,  $\tilde{X}_i(t)$ , is that it represents  
 244 the motion characteristics independent of its location.

245 We generate two feature vectors from  $\tilde{X}_i(t)$ ,  $\mathbf{v}_0$  and  $\mathbf{v}_1$  as the velocity  
 246 term. By randomly choosing four time instances as cutting points,  $t_c (c =$   
 247  $0, \dots, 3; t_c < t_{c+1})$ , we extract

$$\mathbf{v}_0 = \bar{\mathbf{x}}(t_1) - \bar{\mathbf{x}}(t_0), \quad (5)$$

$$\mathbf{v}_1 = \bar{\mathbf{x}}(t_3) - \bar{\mathbf{x}}(t_2). \quad (6)$$

248 Namely, we sample two random velocities,  $\mathbf{v}_0$  and  $\mathbf{v}_1$ , along a trajectory  
 249 by choosing two points per velocity, see Figure 4 (a). This operation of  
 250 cutting a trajectory at four points is motivated by the observation that four  
 251 dynamical models per gait cycle is a reasonable choice in a probabilistic  
 252 decomposition human gait (Bregler, 1997) where coherent motion is used as  
 253 low-level primitives. Note that  $t \geq t_c \geq t - T$ . We then define our features,  
 254  $f_s$  and  $f_d$ , by the distance and the inner product of scaled versions of the two  
 255 vectors:

$$f_s = \|a_0\mathbf{v}_0 - a_1\mathbf{v}_1\|, \quad (7)$$

$$f_d = \langle b_0\mathbf{v}_0, b_1\mathbf{v}_1 \rangle, \quad (8)$$

256 where  $a_i$  and  $b_i, i = 0, 1$  are random coefficients in  $(0, 1)$ . Different features  
 257  $f_s$  and  $f_d$  are generated by sampling values for the coefficients  $a_i$  and  $b_i$ , as  
 258 well as the cutting points,  $t_c (c = 0, \dots, 3)$ , of the trajectory.

259

### 260 3.1.2. Learning Using Random Samples

261 We obtain training data by manually annotating points corresponding  
 262 to foot regions in a video, and then extracting probabilistic trajectories,  
 263  $X_i(t)$  of length  $T$ , as random subgraphs which stem from the annotated  
 264 corners. Extracting trajectories for each corner in each frame, we obtain a  
 265 large number of training data. Training is performed separately for each tree  
 266 using a random subset of the training data.

267 We recursively split the training data at each node, using the standard  
 268 method involving information gain (Breiman, 2001). Namely, we plot the  
 269 responses of randomly selected features in a histogram and learn a threshold,  
 270  $\theta$ , which gives the maximum information gain. A node becomes a leaf node  
 271 where the information gain is below a threshold value. At each leaf node, the  
 272 class distribution of foot/non-foot is computed from the number of instances  
 273 that reach the node. See Figure 5 for an example of our decision forest.

274 We annotate the ground truth data of feet with a tag of left/right foot so  
 275 that they can be directly used for training the decision forest in the second  
 276 stage. It should be noted that those points that can be associated with both  
 277 feet are also annotated with equal probabilities of being on left/right foot.  
 278 See Figure 6 for an example of ground truth labels.

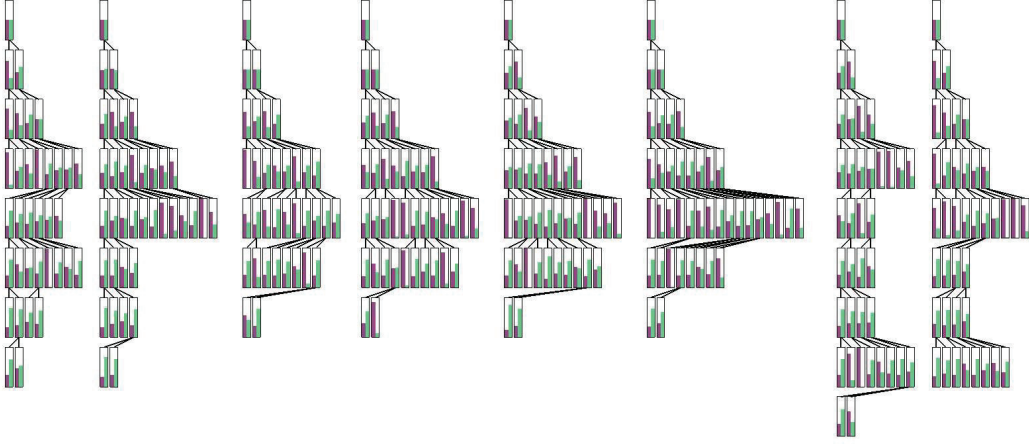


Figure 5: **Example overview of the decision forest** (of the second stage,  $F = 8$ ). The learned class distributions of foot/non-foot are displayed for each node as a histogram.

279 *3.1.3. Selection of Trajectories*

280 We classify candidate trajectories with a decision forest (Breiman, 2001;  
 281 Geurts et al., 2006) which is an ensemble of  $F$  decision trees. Each tree  
 282 examines all input trajectories,  $\tilde{X}_i(t)$ . Given an input trajectory at the root  
 283 node, each decision tree recursively branches left or right down to the leaf  
 284 nodes according to the feature response,  $f_s$  and  $f_d$  in (8), of a learned function  
 285 at each non-leaf node. At the leaf nodes, we obtain the class distributions  
 286 of foot/non-foot. The output from  $F$  decision trees is averaged to select  
 287 candidate trajectories.

288 *3.2. Pairwise Classification of Walking Motion*

289 Given that a corner  $p_i(t)$  is selected as a candidate point in the first  
 290 stage, we pick those  $p_u(t) (u \neq i)$  which are located in the neighborhood of  
 291  $p_i(t)$  and examine how their probabilistic trajectories,  $\tilde{X}_i(t)$  and  $\tilde{X}_u(t)$ , are  
 292 spatio-temporally correlated.

293 *3.2.1. Features for Directional Correlation*

294 Although the trajectory,  $X_i(t)$ , is three-dimensional, when walking in a  
 295 straight line, the trajectory lies approximately in a 2D plane. If a set of  
 296 two candidate trajectories,  $X_i(t)$  and  $X_u(t)$ , arises from walking motion of

297 two feet, the orientations of their 2D planes in 3D space should be close to  
 298 each other because of the consistency of a pair of step motions (Hoffman and  
 299 Flinchbaugh, 1982). Based on this observation, we compute the covariance  
 300 matrices,  $C_i$ , of  $\tilde{\mathbf{x}}(\tau)$ ,  $\tau = t, \dots, t - T + 1$ , and the eigenvector,  $\xi_i \in \mathbf{R}^2$ ,  
 301 corresponding to the greatest eigenvalue so that  $\xi_i$  represents the principal  
 302 direction of  $\tilde{X}_i(t)$  along its 2D plane, see Figure 4 (b). Analogously  $\xi_u$  is  
 303 computed for  $\tilde{X}_u(t)$ .

304 We expect  $\xi_i$  and  $\xi_u$  to be approximately parallel, their directions should  
 305 be both close to the walking direction. For most of the gait cycle the vector  
 306 connecting the two front points on the trajectory  $\mathbf{x}_{iu}(t) = \mathbf{x}_i(t) - \mathbf{x}_u(t)$  can  
 307 be used as an approximation for this direction. We compute a feature vector  
 308 containing inner products,  $\mathbf{c} \in \mathbf{R}^3$ ,

$$\mathbf{c} = \begin{bmatrix} \|\langle \xi_i, \xi_u \rangle\| \\ \|\langle \xi_i, \mathbf{x}_{iu}(t) \rangle\| \\ \|\langle \xi_u, \mathbf{x}_{iu}(t) \rangle\| \end{bmatrix} \quad (9)$$

309 and a random vector  $\phi \in \mathbf{R}^3$ ,  $\|\phi\| = 1$ , so that

$$f_o = \langle \phi, \mathbf{c} \rangle. \quad (10)$$

### 310 3.2.2. Features for Walking Phase Correlation

311 Importantly, we design a feature based on the fact that trajectories from  
 312 a pair of feet are out of phase with each other, alternating in a cyclic manner  
 313 with dynamic and static phases. This means that one foot is mainly in the  
 314 dynamic phase while the other is in the static phase. Since one has nearly  
 315 zero velocity during most of the cycle, we can expect the dot product of their  
 316 velocity vectors, after proper rectification, to be also close to zero. For this  
 317 purpose we consider the trajectory,  $X_i(t)$ , in terms of velocity by generating  
 318 a vector

$$Y_i(t) = [\mathbf{y}(t), \mathbf{y}(t-1), \dots, \mathbf{y}(t-T+2)]^\top \in \mathbf{R}^{2(T-1)} \quad (11)$$

319 where  $\mathbf{y}(\tau) = \mathbf{x}(\tau) - \mathbf{x}(\tau-1)$ ,  $\tau = t, \dots, t - T + 2$ . We convert each  $\mathbf{y}(\tau)$  to  
 320  $\check{\mathbf{y}}(\tau)$  by projecting it to the axis of  $\xi_i$ . Thus, the rectified velocity vector is

$$\check{Y}_i(t) = [\check{\mathbf{y}}(t), \check{\mathbf{y}}(t-1), \dots, \check{\mathbf{y}}(t-T+2)]^\top. \quad (12)$$

321 Rather than simply taking the inner product of the entire  $\check{Y}_i(t)$  and  $\check{Y}_u(t)$ ,  
 322 which would result in a scalar, we compute their piecewise dot products. By



Figure 6: **Ground truth labels:** Corners detected inside the circles are annotated as being on a foot. We use an in house annotation tool which accelerates the process by allowing probabilistic labelling.

323 cutting each of  $\check{Y}_i(t)$  and  $\check{Y}_u(t)$  into  $l$  pieces at common fixed cutting points,  
 324  $t_c (c = 0, \dots, l - 2; t_c > t_{c+1})$ , we acquire a vector

$$\mathbf{q} = [\langle \check{Y}'_i(t), \check{Y}'_u(t) \rangle, \dots, \langle \check{Y}'_i(t_{l-2}), \check{Y}'_u(t_{l-2}) \rangle]^\top \in \mathbf{R}^l, \quad (13)$$

325 where  $\check{Y}'_i(t_c)$  represents a portion of  $\check{Y}_i(t)$  starting at  $t_c$ . We then define a  
 326 phase feature  $f_p$  as the inner product of  $\mathbf{q}$  with a random vector,  $\psi \in \mathbf{R}^l$   
 327 where  $\|\psi\| = 1$ :

$$f_p = \langle \psi, \mathbf{q} \rangle. \quad (14)$$

328 We choose to use  $l = 5$ , again assuming that the four dynamical models per  
 329 gait are well covered in the trajectories.

### 330 3.2.3. Final Detector Output

331 The output from the second decision forest consists of a set of hundreds  
 332 of feature pairs along with their probabilities of being a pair of feet (see  
 333 bottom-right in Figure 7 for an example). In order to extract a major pair  
 334 from this set, we run mean-shift clustering and take the average of the most  
 335 probable cluster as the final estimate.

336 Note that the cost for the algorithm including the classification and this  
 337 clustering step could vary depending on the number of candidate trajectories  
 338 that are selected in the first stage.



Figure 7: **Results on Sequence I** Detection results superimposed on the input frames (from left to right, 150 frames between each view). Top: Extracted corner points and candidate points after the first stage are shown in green, the rejected points in purple, and points with trajectories of insufficient length in black. Middle: Pairs of corners extracted as feet in the second stage shown as green line segments. Bottom: Final detection after running mean-shift. Right: Example of all possible pairs between pre-selected corners at stage 1. Purple line segments are those rejected in stage 2.

#### 339 4. Experiments

340 In order to obtain training data we made manual annotations in real  
 341 training images. Figure 6 shows how we acquire them in a sequence. The  
 342 two circles indicate areas where corners detected inside are annotated as being  
 343 on foot in the current frame. Corners on the right foot and the left foot are  
 344 annotated separately by brown and yellow circles, respectively. The smaller  
 345 circles indicate the annotated locations of the feet in other frames of the image  
 346 sequence. As inputs, we captured video sequences (resolution  $1280 \times 720$   
 347 pixels at 60 fps) in which a person walks in seven different directions as well  
 348 as other sequences including different persons walking at different speed.

349 Figure 7 illustrates the performance of the proposed classification on a  
 350 sequence of 350 frames. The selected candidates of the first stage and the  
 351 classified pairs in the second stage are shown in green, in the top and the  
 352 middle rows, respectively. Although there are some connections between  
 353 corners for example on arms as their motion is similar to feet (see the bottom-  
 354 right picture in a larger scale), the connections between feet are generally  
 355 dominant, and the final detection results are shown in red in the bottom  
 356 row. The algorithm currently runs at 2-5 frames per second.

357 Figure 8 shows detection in a 400-frame sequence of two pedestrians cross-



Figure 8: **Results on Sequence II.** *Detection of bipedal motion of two people walking across the scene from opposite directions (from top to bottom). Left: Candidate points after the first stage are shown in green, rejected points in red, points with short trajectories in black. Middle: Candidate pairs after the second stage shown as green line segments. Right: Final detection result of pairs of feet after running mean-shift shown as red line segments.*



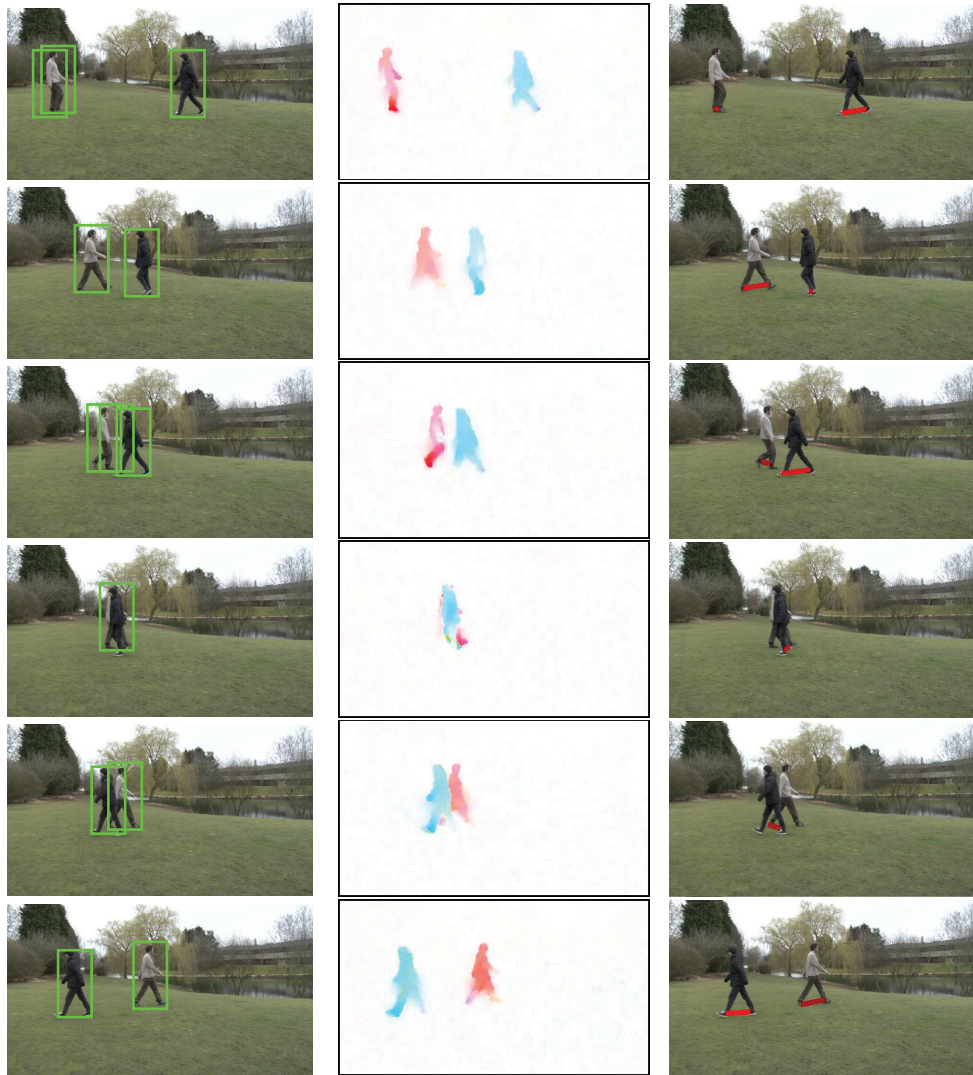


Figure 9: **Pedestrian detection on Sequence II by motion-based technique.** *Detection of two pedestrians walking across the scene from opposite directions by a sliding window search with histograms of flow (HOF) and HOG descriptors. See texts for details. Left: Final detection result shown with green bounding boxes. Middle: Color coded optic flow used for computing histograms of flow. Right: Pairs of feet detected by the proposed method: the same results that are in Figure 8 (for side-by-side comparison).*

Table 1: **The detection rates (%)**. *Top: The detection rates of pairs of feet for the sequence including two pedestrians. See text for the definition of  $r_{simple}$  and  $r_{pure}$ . Middle: The detection rates of pedestrians by a sliding window search with the HOF and HOG descriptors (Walk et al., 2010). Bottom: The detection rates of pairs of feet using the annotations as the ground truth. (The threshold distance is set to 30 pixels.)*

Proposed	$r_{simple}$	$r_{pure}$
Person A (front)	90.6	83.4
Person B (back)	78.4	69.1

HOF+HOG	$r_{simple}$	$r_{pure}$
Person A (front)	92.4	83.6
Person B (back)	83.4	67.2

Proposed	Example 1	Example 2
Single person	68.9	66.0

358 ing the scene. Candidate of a set of feature (trajectory) pairs as the output  
 359 from the second decision forest are shown as green line segments in the sec-  
 360 ond column. Note that we can observe two obvious modes corresponding to  
 361 two pedestrians for which we run mean-shift clustering and take the average  
 362 of the most probable cluster(s). This example shows that we can select more  
 363 than one major mode at this stage to determine the final estimate when  
 364 dealing with multiple targets.

365 Table 1 (top) further shows the detection rates for each of the two pedes-  
 366 trians;  $r_{simple}$  is based on a simple count of successful cases where a mode  
 367 connecting the two feet is detected whereas  $r_{pure}$  indicates a similar rate but  
 368 excluding the cases when an extra mode is also detected by mistake for in-  
 369 stance between an arm and the background. The overall detection rates are  
 370 lower for Person B that is walking in behind and occluded by Person A in  
 371 part of the sequences. During this crossing phase only one pair of feet is  
 372 detected, but subsequently both are detected again correctly. The supple-  
 373 mentary video demonstrates the performance of detections.

374 For a comparison, we have also applied a state-of-the-art motion-based  
 375 pedestrian detector with histograms of flow (HOF) descriptor<sup>2</sup> (Dalal et al.,

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<sup>2</sup>The exact descriptor we implemented is called IMHd2 (Walk et al. CVPR10) which is

376 2006; Walk et al., 2010) to Sequence II in a framework of sliding window  
 377 search; Figure 9 shows some examples of the results with green bounding  
 378 boxes as well as the color coded optic flow (Werlberger et al., 2009) which  
 379 was the basis for computing HOF features. The results demonstrate that  
 380 the motion-based pedestrian detection also performs well generally, missing  
 381 the target only during the crossing phase. By the nature of sliding window  
 382 search, however, multiple detections could appear for each target (e.g. in the  
 383 top row) although a non-maximum suppression has been performed. Also,  
 384 a spurious detection is observed when two targets are close to each other  
 385 (see the third row). Note that in the same frame their feet are detected  
 386 properly by the proposed approach. Nevertheless, when two people are half  
 387 overlapping (the fifth row), the clustering process after the feet detection is  
 388 confused (see also the corresponding row of Figure 8 for the raw detections)  
 389 while the pedestrian detection is performing stably.

390 Another difference that should be addressed is the location of detection;  
 391 some ambiguity is inherent in detections with a sliding window search, which  
 392 is not considered as a problem when evaluated by the intersection-over-union  
 393 measure with annotations, whereas the proposed method directly indicates  
 394 quite precise positions of feet in the given image. We will discuss the impor-  
 395 tance of this issue in terms of an application in Section 6.

396 Table 1 (middle) shows the detection rates for each of the two pedestri-  
 397 ans computed for Sequence II by the sliding window search with HOF and  
 398 HOG descriptors (Walk et al., 2010). Analogously to the case with our feet  
 399 detection,  $r_{simple}$  refers to a simple count of successful pedestrian detections  
 400 in terms of intersection-over-union measure whereas  $r_{pure}$  indicates a simi-  
 401 lar rate but excluding the cases when wrong detection(s) also occurred by  
 402 mistake. As was the case with feet detection, the overall detection rates are  
 403 lower for Person *B* than Person *A* mainly due to the crossing phase. The per-  
 404 formance is somewhat comparable to the proposed feet detection although it  
 405 is not possible to deduce superiority of one against the other; the IMHd2 is  
 406 computed for the entire pedestrian regions at a time but uses optic flow just  
 407 based on two frames whereas the feet detection uses only local information  
 408 as few as two feature points but for 60 frames. If we make a simple com-

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combined with HOG features. We used the TUD-MotionPairs dataset (Wojek et al., 2009)  
 for training the model as suggested in (Walk et al., 2010), and the histogram intersection  
 kernel SVM (Maji et al., 2008) for the classifier.

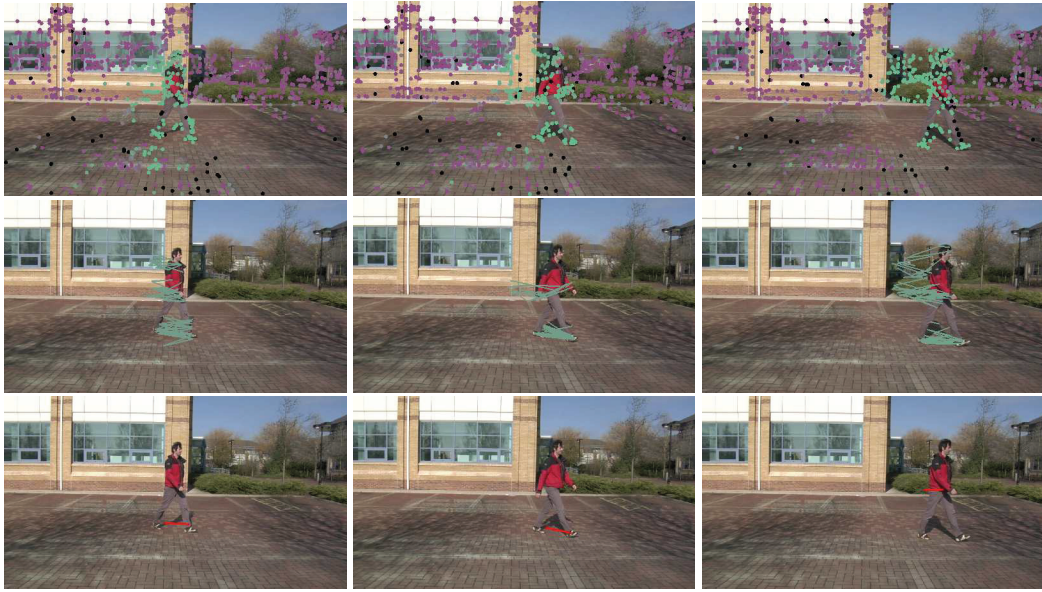


Figure 10: **Results on Sequence III.** Feet are correctly detected in the first frames (left, middle), but in the last frame (right) a detection in the arm region occurs due to very similar motion. Top: Candidate points after the first stage are shown in green, rejected points in purple, points with short trajectories in black. Middle: Candidate pairs after the second stage are green line segments. Bottom: Final detection after running mean-shift.

409 parison for Person A,  $r_{simple}$  with IMHd2 is a little higher than that with  
410 the proposed feet detection (see Table 1 (top)) while  $r_{pure}$  being at the same  
411 level, implying an equivalent false positive ratio.  $r_{simple}$  with IMHd2 for Per-  
412 son B is also higher than that with the feet detection, but the score of  $r_{pure}$   
413 drops significantly to the level that is lower than the proposed feet detection,  
414 reflecting more cases of spurious detections involved in the window search.

415 In order to evaluate the detection rate in a more strict sense, for an  
416 outdoor sequence with one pedestrian of 595 frames, we compute the error  
417 as distance between the detected pairs and the annotated pairs. For this  
418 the correspondences between the two pairs is found and the error defined  
419 as the average distance between corresponding points. The average error is  
420 less than a threshold distance<sup>3</sup> from the ground truth for 410 frames. For  
421 another sequence, the distance was below the same threshold for 310 frames  
422 out of 470 input frames. Approximately half of the error cases were due to  
423 incorrect detection of arm motion. See Table 1 (bottom) for the summary of  
424 the detection rates considering the distances to the annotations.

## 425 5. Discussion

### 426 5.1. Failure Case

427 Figure 10 shows an example sequence of 500 frames where outliers become  
428 more dominant and the final detection is no longer at the foot location.  
429 Although pairs are detected on the feet (left), a number of pairs remain  
430 candidates as the result of classification stages. Outliers include pairs of  
431 points on the arms as well as pairs between the body and the background.  
432 Through a detailed analysis typical cases have been found where a trajectory  
433 is first on body but taken over the background, or vice versa. Those pairs  
434 tend to cluster and take over the final signal as being from feet after a few  
435 seconds. However, one possible approach to avoid those cases will be to  
436 perform further careful training by using such instances as negative examples.

### 437 5.2. Occlusions and Crowd

438 One of the benefits of our new representation is that it tolerates inherent  
439 ambiguity due to occlusion which occurs internally to a target. However,  
440 external occluding object/target would be a cause of error, just as in many

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<sup>3</sup>We set it to be 30 pixels in this evaluation.

441 existing algorithms, as shown in Figure 8 for the case with two people; de-  
 442 pending on the phase of the overlap the output of the final clustering process  
 443 could be a pair of feet of the frontal person, or a pair of feet of two nearby  
 444 people whose motion happens to be correlated in the similar way as that of  
 445 a real pair of feet does. The stability against more precise classification is  
 446 therefore in working progress.

447 The current approach will be confused with further crowded people, being  
 448 not suitable to such a situation. For dealing with complicated crowded se-  
 449 quence, other representations such as particle trajectories (Wu et al., 2010)  
 450 have been proposed which are designed for anomaly detection while pro-  
 451 ducing some representative trajectories of a crowd. On the other hand, our  
 452 approach will help when the position of feet in given images need be detected  
 453 explicitly.

### 454 5.3. Moving Camera

455 Moving cameras generally pose significant challenges for recognising mo-  
 456 tion due to the changes in the field of view. Although we have assumed a  
 457 static camera to capture input sequences, future work will be directed to the  
 458 case of a moving camera. In order to tackle this problem, it will be necessary  
 459 to separate the global camera motion and the local object motion somehow  
 460 in the acquired frames. Figure 11 shows trajectories for the cases with both  
 461 stationary and moving camera. The space-time volume is displayed so that  
 462 the time-axis is along the vertical direction. In the case of a stationary cam-  
 463 era, trajectories connecting background corners are vertically aligned. On the  
 464 other hand, trajectories viewed by a moving camera exhibit more variation.  
 465 However, for sufficiently smooth camera motion the trajectories of points on  
 466 the feet are still recognizable, and therefore we believe that it is possible to  
 467 eliminate the dominant camera motion.

468 One way to deal with such variations is to generate a rectified velocity  
 469 vector in (12) so as to cancel the possible camera motion; given a velocity  
 470 vector,  $Y_i(t)$ , as in (11) we can compute the rectification by

$$\check{\mathbf{y}}(\tau) = \hat{\mathbf{y}}(\tau) - \min_{\tau} \hat{\mathbf{y}}(\tau) \quad (15)$$

471 where  $\hat{\mathbf{y}}(\tau)$  is obtained by taking the absolute value of each element of re-  
 472 sulting vector after the projection.

473 Other possibilities are to employ a global motion compensation step sim-  
 474 ilar to (Mikolajczyk and Uemura, 2008) or the motion features computed

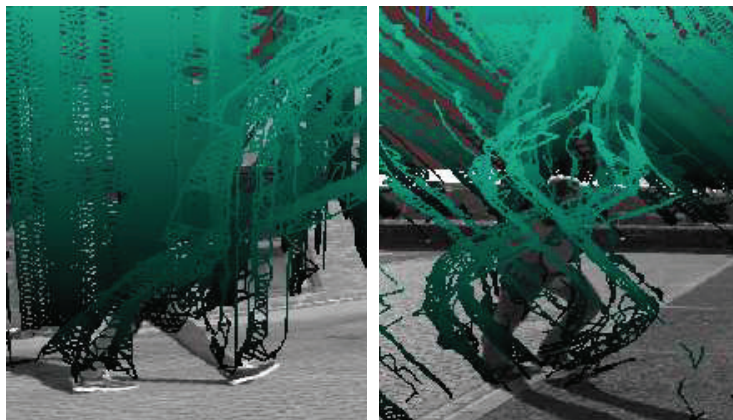


Figure 11: **Static and moving camera.** *Point trajectories in the case of a stationary camera (left) and a moving camera (right).*

475 from 3D trajectories introduced in (Brostow et al., 2008). Also, a promis-  
 476 ing approach to handle the moving camera is to decompose the trajectories  
 477 into their camera-induced and object-induced componens based on low rank  
 478 optimization as recently suggested in (Wu et al., 2011).

## 479 6. Conclusion

480 We have introduced a new algorithm for detecting bipedal motion from  
 481 point trajectories. In particular, we proposed to use the fact that trajec-  
 482 tories from two feet are spatio-temporally correlated. To this end, we have  
 483 introduced (i) the notion of *probabilistic trajectories*, (ii) the pairwise anal-  
 484 ysis of trajectories for detecting their correlation, (iii) the design of efficient  
 485 features for a two-stage decision forest classifier, and (iv) potential extensions  
 486 to deal with camera motion. To the best of our knowledge this is the first  
 487 attempt to recognize walking motion by way of investigating correlation of  
 488 point trajectories.

489 The strategy in this paper was to retain the uncertainty in the track  
 490 associations and let the classifier handle this uncertainty. However, other  
 491 advanced methods for association could result in less ambiguous trajectories  
 492 and thus allow the features to be more discriminative. Although our method  
 493 currently assumes little variation in walking speed, it will be also useful in  
 494 the future work to model the period of a walk cycle (Cutler and Davis, 2000;

495 Laptev et al., 2005) and to identify each phase of walking motion.

496 The original goal of the work is to replicate the ability to recognize motion  
497 by the effect of *kinetic depth* using tracked points on a pair of feet. In  
498 terms of applications, we have designed the method to be used as a module  
499 of pedestrian detection system, where feet detection helps to measure the  
500 distance to the target since the 2D location in the image can be directly  
501 mapped to 3D distance given a calibrated camera. This function would be  
502 especially useful for an automotive system to measure the time-to-contact in  
503 the area of monocular surveillance given the driving speed (Enzweiler et al.,  
504 2008). State-of-the-art methods for pedestrian detection based on a window  
505 search return a bounding box as the detected result, but the bottom end of  
506 the bounding box does not necessarily coincide with the precise 2D location  
507 of feet. Thus, we point out that the proposed motion-based technique may  
508 well complement appearance-based methods such as in (Dalal and Triggs,  
509 2005) in the context of pedestrian detection.

## 510 **Appendix A. The Canonical Form of Trajectories**

511 The canonical form,  $\tilde{X}_i(t)$ , of a trajectory  $X_i(t)$ , is computed as

$$\tilde{X}_i(t) = X_i(t) - \bar{X}_i(t) \quad (\text{A.1})$$

512 where  $\bar{X}_i(t) = [\bar{\mathbf{x}}(t), \dots, \bar{\mathbf{x}}(t - T + 1)]^\top$  and

$$\bar{\mathbf{x}}(\tau) = \frac{1}{T-1} [(t-\tau) \mathbf{x}(t-T+1) + (\tau-t+T-1) \mathbf{x}(t)]. \quad (\text{A.2})$$

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