Engineering Tripos Part IIB FOURTH YEAR

Module 4F12: Computer Vision

Examples Paper 3

Straightforward questions are marked †
Tripos standard (but not necessarily Tripos length) questions are marked $*$

1. † *The stereo correspondence problem*

   What matching constraints can be used to find point correspondences in stereoscopic vision? What is meant by an epipole and the epipolar constraint?

2. † *Stereo and orthographic projection*

   A friend reading architecture considers the stereo reconstruction problem and suggests that the 3D position of a point can be determined by using the left image to obtain the X and Y world coordinates and the right image to find the depth (Z coordinate). Comment on this suggestion.

3. $*$ *Epipolar geometry*

   (a) A stereoscopic pair of cameras is arranged symmetrically with the two optical axes coplanar and making an angle of 45°. Sketch the families of epipolar lines in the left and right images. Assume both cameras have the same focal length f. (Hint: before sketching epipolar lines in the left image, find the projection, onto the left image plane, of the right camera's optical centre.)

   (b) Now solve part (a) algebraically. That is, given a point (x', y') in the right image, find the equation of the line in the left image on which any corresponding point (x, y) must lie (and vice-versa).

4. $*$ *Triangulation with rays*

   Consider a parallel stereo pair (image planes of the two cameras are aligned) with both cameras having the same focal length and with a baseline of d:

   $$\mathrm{R} = \mathrm{I}\,, \quad \mathbf{T} = [\, -d \;\; 0 \;\; 0 \,]^T$$

   Show that the depth along the optical axis in the first view, Z_c, can be recovered directly from the disparity between the image positions in the left and right images: $Z_c = df/(x - x')$.

5. $*$ *Uncalibrated stereo and the fundamental matrix*

If $\mathbf{w}$ and $\mathbf{w}'$ are corresponding pixels in left and right images, show that they are related by

$$[\, u' \;\; v' \;\; 1 \,]\, F \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = 0$$

where F is the fundamental matrix. Show how F is related to the *isometry*[1] between the two camera coordinate systems and the *intrinsic* camera parameters.

Explain how the fundamental matrix can be used to solve for the epipoles in the two views, and why the maximum rank of F is 2. How many point correspondences are required to estimate F?

6. *Affine fundamental matrix*

Derive expressions for the left (u, v) and right (u', v') pixel positions of a point in space viewed through *weak* perspective cameras. By eliminating the world coordinates or by showing that the 4 image coordinates (in 3 unknowns) are not independent, show that the pixel coordinates are related by

$$[\, u' \;\; v' \;\; 1 \,]\, F_A \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = 0$$

where F_A is the affine fundamental matrix which has maximum rank 2 and can be expressed in the form

$$F_A = \begin{bmatrix} 0 & 0 & a \\ 0 & 0 & b \\ c & d & 1 \end{bmatrix}$$

Show that epipolar lines under weak perspective are parallel.

Suitable past Tripos questions: 2001-2014 Q4 and 2015-2019 Q3

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[1] An "isometry" is a rigid body transformation.