Stereo Vision Algorithms

- Epipolar Constraints
- Ordering Constraint
- Figural Continuity
- Dynamic Programming (Dijkstra)

Virtual reality

(Anandan, Criminisi, Kang, Szeliski, Uyttendaele, Microsoft Research)
**View interpolation**

![Diagram showing left and right camera images, stereo reconstruction, and cyclopean ground truth]

**Stereo Problems:**

Parallax: \( L(x+d) = R(x) \) \[ \hat{x} = (x, y) \]

1) estimate stereo disparity
2) estimate 'cyclopean' intensity

Estimate parallax: \( \hat{d} = \arg \max p(d \mid L, R) \)

Cyclopean image: \( \bar{I} = I(d, L, R) \)
Stereo Ambiguity Problem

? How many potential matches for \( N \times N \) image?

Human stereo vision
Epipolar geometry
(tutorial by Andrea Fusiello)

Matching space

? Why only lower triangle
Stereo disparity on epipolar lines

Intensity on left epipolar line:
\[ L = \{ L_m, \ m = 0, \ldots, N \} \]

Intensity on left epipolar line:
\[ R = \{ R_n, \ n = 0, \ldots, N \} \]

Matched pair
\[ x_k = (m, n) \]

has "disparity"
\[ d_k = m - n > 0 \]

Matching path
\[ x = (x_1, \ldots, x_k, \ldots, x_K) \]

Epipolar matching and stereo disparity

? Legal paths
Double Nail Illusion

Ordering constraint

Nail illusion
Epipolar matching as optimal path finding

(Ohta & Kanade, 1985; Cox, Hingorani & Rao, 1996)

Min cost path: \( \min_d F(d, L, R) \)

\[
F(d, L, R) = \sum_k g(d_k, d_{k-1}) + \sum_k f_k(d_k; L, R)
\]

\[
g(d_k, d_{k-1}) = \begin{cases} 
0 & \text{if } d_k = d_{k-1} \\
\gamma & \text{if } d_k = d_{k-1} \pm 1 \\
\infty & \text{otherwise}
\end{cases}
\]

\[
f_k(d_k; L, R) = \ ??
\]

Epipolar match: minimal path
Matching function

Epipolar matching as optimal path finding

Min cost path: \( \min_d F(d, L, R) \)

\[
F(d, L, R) = \sum_k g(d_k, d_{k-1}) + \sum_k f_k(d_k; L, R)
\]

“three-move”

Dijkstra algorithm
Example: Dijkstra algorithm

Take $\gamma=1$
Cyclopean Coordinates
(Marr & Poggio, 1976; Belhumeur 1996)

- Cyclopean projection
- Rendering from L/R match

Gaze correction: virtual cyclopean camera

Dijkstra solution (1D fps)

?what more
Figural continuity constraints

Left Image

Figural continuity constraints

Right Image
Markov chains

\[ d = (d_1, d_2, \ldots, d_t, \ldots, d_T) \]

1\textsuperscript{st} order Markov chain

2\textsuperscript{nd} order Markov chain

.. Markov processes

- Markov Random Field
- Intractable cf. Markov chain

\[ d_n, d_{n+1}, d_{n,k}, d_{n,k+1} \]

DP ruined!! -- solution?

1. Raster scan DP
2. Min Cut
Applying figural continuity constraints

(Cox et al., Computer Vision and Understanding, 1996)