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Following Cusps

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Abstract. It is known that the deformation of the apparent contours of a surface under perspective projection and viewer motion enable the recovery of the geometry of the surface, for example by utilising the *epipolar parametrization*. These methods break down with apparent contours that are singular i.e., with *cusps*. In this paper we study this situation and show how, nevertheless, the surface geometry (including the Gauss curvature and mean curvature of the surface) can be recovered by *following the cusps*. Indeed the formulae are much simpler in this case and require lower spatio-temporal derivatives than in the general case of nonsingular apparent contours. We also show that following cusps does not by itself provide us with information on viewer motion.

1. Introduction

For smooth curved surfaces an important image feature is the profile or apparent contour. This is the projection of the locus of points on the surface which separates the visible and occluded parts. See Fig. 1. Under perspective projection this locus—the critical set or contour generator—can be constructed as the set of points on the surface which are touched by rays through the projection centre. The contour generator is dependent on the local surface geometry (via tangency and conjugacy constraints) and the viewpoint. Each viewpont will generate a different contour generator with the contour generators 'slipping' over the visible surface under viewer motion.

The family of contour generators generated under continuous viewer motion can be used to represent the visible surface. Giblin and Weiss (1987) and Cipolla and Blake (1992) have shown how the spatio-temporal analysis of deforming image apparent contours (*profiles*) enables computation of local surface curvature along the corresponding contour generator (*critical sets*) on the surface, assuming viewer motion is known. To perform the analysis, however, a spatio-temporal parametrization of image-curve motion is needed, but is underconstrained. The *epipolar* parametrization is most naturally matched to the recovery of surface curvature. In this parametrization (for both the spatio-temporal image and the surface), *correspondence* between points on successive snapshots of an apparent contour is set up by matching along epipolar lines. Namely the corresponding ray in the next viewpoint (in an infinitesimal sense), is chosen so that it lies in the epipolar plane defined by the viewer's translational motion and the ray in the first viewpoint. The parametrization leads to simplified expressions for the recovery of depth and surface curvature from image velocities and accelerations and known viewer motion. It is especially suited to the recovery of surface geometry by an active explorer making deliberate viewer motions around an object of interest and it has been successfully implemented in various systems