Short Papers

Estimating the Fundamental Matrix via Constrained Least-Squares: A Convex Approach

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Abstract—In this paper, a new method for the estimation of the fundamental matrix from point correspondences is presented. The minimization of the algebraic error is performed while taking explicitly into account the rank-two constraint on the fundamental matrix. It is shown how this nonconvex optimization problem can be solved avoiding local minima by using recently developed convexification techniques. The obtained estimate of the fundamental matrix turns out to be more accurate than the one provided by the linear criterion, where the rank constraint of the matrix is imposed after its computation by setting the smallest singular value to zero. This suggests that the proposed estimate can be used to initialize nonlinear criteria, such as the distance to epipolar lines and the gradient criterion, in order to obtain a more accurate estimate of the fundamental matrix.

Index Terms—Stereo vision, fundamental matrix, convex optimization, linear matrix inequality.

1 Introduction

The computation of the fundamental matrix existing between two views of the same scene is a very common task in several applications in computer vision, including calibration and reconstruction [9], visual navigation, and visual servoing. The importance of the fundamental matrix is due to the fact that it represents succinctly the epipolar geometry of stereo vision. Indeed, its knowledge provides relationships between corresponding points in the two images. Moreover, for known intrinsic camera parameters, it is possible to recover the essential matrix from the fundamental matrix and, hence, the camera motion between the views [4].

Several techniques have been developed for the estimation of the fundamental matrix from point correspondences, like the linear criterion, the distance to epipolar lines criterion, and the gradient criterion (see, e.g., [8], [11], [7], [13]). The first one is a least-squares technique minimizing the algebraic error. This approach has proven to be very sensitive to image noise and it does not consider the fact that the rank of the fundamental matrix must be equal to 2. The other two techniques take into account the rank constraint and minimize a more indicative distance, the geometric error, in the seven degrees of freedom of the fundamental matrix. This results in nonconvex optimization problems that present local solutions in addition to the global ones. Hence, the solution found via numerical procedures is affected by the choice of the starting point of the minimization algorithm [8]. Generally, this point is chosen as the estimate provided by the linear criterion and forced to be singular by setting the smallest singular value to zero, but this choice does not guarantee to find the global minimum.

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In this paper, we present a new method for the estimation of the fundamental matrix. It consists of a constrained least-squares technique in which the rank condition on the matrix is ensured by the constraint. In this way, we impose the singularity of the matrix a priori instead of forcing it after the minimization procedure as in the linear criterion. Our aim is twofold: first, we show how this optimization problem can be solved avoiding local minima. Second, we provide experimental results showing that our approach leads to a more accurate estimate of the fundamental matrix. In order to find the global minimum, we start by showing how this problem can be addressed as the minimization of a rational function in two variables. Then, we reformulate the minimization problem so that it can be tackled by recently developed convexification techniques [2], which guarantee that local optimal solutions are avoided and only the global one is found.

The same problem has been studied by Hartley [7], who provided a method for minimizing the algebraic error ensuring the rank constraint, which requires an optimization over two free parameters (position of an epipole). However, the resulting optimization procedure does not guarantee to avoid local minima in the general case. Another approach has been proposed by Faugeras [5] in which the fundamental matrix is obtained via the solution of a third degree homogeneous equation. Nevertheless, this method does not guarantee minimization of the algebraic error subject to the rank constraint.

2 Preliminaries

First, let us briefly introduce the notation used in this paper: I_n is the $n \times n$ identity matrix; A^T is the transpose of A; $\|u\|_{2,W} = \sqrt{u^T W u}$ is the weighted Euclidean norm of u; $\mathrm{adj}(A)$ is the adjoint of A; $\lambda_M(A)$ denotes the maximum real eigenvalue of A.

Given a pair of images, the fundamental matrix $F \in \mathbb{R}^{3\times 3}$ is defined as the matrix satisfying the relation

$$u'^T F u = 0 \quad \forall u', u, \tag{1}$$

where $u', u \in \mathbb{R}^3$ are the projections expressed in homogeneous coordinates of the same 3D point in the two images. The fundamental matrix F has seven degrees of freedom being defined up to a scale factor and being singular [5]. The *linear criterion* for the estimation of F is defined as

$$\min_F \sum_{i=1}^{n} \left(u_i'^T F u_i \right)^2, \tag{2}$$

where n is the number of observed point correspondences. In order to obtain a singular matrix, the smallest singular value of the found estimate is set to zero [6]. The *distance to epipolar lines criterion* and the *gradient criterion* take into account the rank constraint using a suitable parameterization for F. The first criterion defines the cost function as the sum of squares of distances of a point to the corresponding epipolar line. The second criterion considers a problem of surface fitting between the data and the surface defined by (1). These nonlinear criteria result in the minimization of weighted least-squares

$$\min_{F:\det(F)=0} \sum_{i=1}^{n} w(F, u_i', u_i) \left(u_i'^T F u_i \right)^2, \tag{3}$$

where $w(F, u_i', u_i)$ is a suitable weighting function [8]. The main problem with these nonlinear criteria is that the cost function in (3) turns out to be nonconvex. Hence, the solutions provided by numerical optimization methods heavily depend on the starting point of the optimization procedure. Experiments show a large difference between results obtained starting from the exact solution and starting from the solution provided by the linear criterion generally used to initialize these minimizations [8].

3 The Constrained Least-Squares Estimation Problem

The problem that we wish to solve can be written as

$$\min_{F:\det(F)=0} \sum_{i=1}^{n} w_i (u_i'^T F u_i)^2, \tag{4}$$

where $n \geq 8$, $w_i \in \mathbb{R}$ are positive given weighting coefficients and the constraint ensures the rank condition on F. Let us introduce $A \in \mathbb{R}^{n\times 8}$, $\operatorname{rank}(A) = 8$, and $b \in \mathbb{R}^n$ such that

$$\sum_{i=1}^{n} w_i (u_i'^T F u_i)^2 = \|Af - b\|_{2,W}^2,$$

where $f = (f_1, \ldots, f_8) \in \mathbb{R}^8$ contains the entries of

$$F = \begin{pmatrix} f_1 & f_4 & f_7 \\ f_2 & f_5 & f_8 \\ f_3 & f_6 & 1 \end{pmatrix}$$

and $W \in \mathbb{R}^{n\times n}$ is a diagonal matrix with positive entries w_i. Since F is defined up to a scale factor, we have set $F_{3,3} = 1$ (this parameterization is not general since $F_{3,3}$ can be zero; however, this problem can be easily overcome by setting to 1 another entry of F, which is not zero for the considered case). Then, (4) can be written as

$$\begin{aligned} &\min_{f,\lambda} \|Af - b\|_{2,W}^2 \\ &\text{subject to} \quad T(\lambda) f = r(\lambda), \end{aligned} \tag{5}$$

where $\lambda = (\lambda_1, \lambda_2) \in \mathbb{R}^2$, $T(\lambda) \in \mathbb{R}^{3\times 8}$, and $r(\lambda) \in \mathbb{R}^3$ are so defined

$$T(\lambda) = \begin{pmatrix} I_3 & \lambda_1 I_3 & \begin{matrix} \lambda_2 & 0 \\ 0 & \lambda_2 \\ 0 & 0 \end{matrix} \end{pmatrix}, \quad r(\lambda) = \begin{pmatrix} 0 \\ 0 \\ -\lambda_2 \end{pmatrix}.$$

The constraint in (5) expresses the singularity condition on F as the linear dependency of the columns of F and, hence, $\det(F) = 0$. In order to solve problem (5), let us observe that the constraint is linear in f for any fixed λ. Therefore, the problem can be solved using the Lagrange's multipliers obtaining the solution $f^*(\lambda) = v - ST(\lambda)P(\lambda)[T(\lambda)v - r(\lambda)]$, where $v = SA^TWb$, $S = (A^TWA)^{-1}$ and $P(\lambda) = \left[T(\lambda)ST^T(\lambda)\right]^{-1}$. Now, it is clear that the minimum J^* of (5) can be computed as

$$J^* = \min_{\lambda} J^*(\lambda) = \min_{\lambda} \|Af^*(\lambda) - b\|_{2,W}^2. \tag{6}$$

Let us calculate $J^*(\lambda)$. Substituting $f^*(\lambda)$ into the cost function, we obtain $J^*(\lambda) = c_0 + \sum_{i=1}^{3} c_i(\lambda)$, where

$$c_0 = b^T W \left(I_n - ASA^T W\right) b,$$

$$c_1(\lambda) = r^T(\lambda) P(\lambda) r(\lambda),$$

$$c_2(\lambda) = -2v^T T^T(\lambda) P(\lambda) r(\lambda),$$

and

$$c_3(\lambda) = v^T T^T(\lambda) P(\lambda) T(\lambda) v.$$

The constrained problem (4) in eight variables is equivalent to the unconstrained problem (6) in two variables. In order to compute the solution J^*, let us consider the form of the function $J^*(\lambda)$. The terms $c_i(\lambda)$ are rational functions of the entries of λ. In fact, let us write $P(\lambda)$ as

$$\begin{aligned} &P(\lambda) = \frac{G(\lambda)}{d(\lambda)}, \quad G(\lambda) = \operatorname{adj}\left[T(\lambda)ST^T(\lambda)\right], \\ &d(\lambda) = \det\left[T(\lambda)ST^T(\lambda)\right]. \end{aligned} \tag{7}$$

Since $T(\lambda)$ depends linearly on λ, we have that $G(\lambda)$ is a polynomial matrix of degree 4 and $d(\lambda)$ a polynomial of degree 6. Straightforward computations allow one to show that $J^*(\lambda)$ can be written as $J^*(\lambda) = \frac{h(\lambda)}{d(\lambda)}$, where $h(\lambda)$ is a polynomial of degree 6 defined as

$$\begin{aligned} h(\lambda) = {} & c_0 d(\lambda) + r^T(\lambda) G(\lambda) r(\lambda) \\ & -2v^T T^T(\lambda) G(\lambda) r(\lambda) + v^T T^T(\lambda) G(\lambda) T(\lambda) v. \end{aligned} \tag{8}$$

Let us observe that the function $d(\lambda)$ is strictly positive everywhere being the denominator of the positive definite matrix $T(\lambda)ST^T(\lambda)$.

4 Problem Solution via Convex Programming

In this section, we present a convexification approach to the solution of problem (4). The technique is based on Linear Matrix Inequalities (LMI) [1] and leads to the construction of lower bounds on the global solution of the polynomial optimization problem (4). More importantly, it provides an easy test to check whether the obtained solution is the global optimum or just a lower bound. From the previous section, we have that $J^* = \min_\lambda h(\lambda) d(\lambda)^{-1}$. Let us rewrite this as

$$\begin{aligned} J^* = {} & \min_{\lambda,\delta} \delta \\ & \text{subject to} \quad \tfrac{h(\lambda)}{d(\lambda)} = \delta, \end{aligned} \tag{9}$$

where $\delta \in \mathbb{R}$ is an auxiliary variable. The constraint in (9) can be written as $y(\lambda, \delta) = 0$, where $y(\lambda, \delta) = h(\lambda) - \delta d(\lambda)$ since $d(\lambda) \neq 0$ for all λ. Hence, J^* is given by

$$\begin{aligned} J^* = {} & \min_{\lambda,\delta} \delta \\ & \text{subject to} \quad y(\lambda, \delta) = 0, \end{aligned} \tag{10}$$

where the constraint is a polynomial in the unknowns λ and δ. Problem (10) belongs to a class of optimizations problems for which convexification techniques have been recently developed [2]. The key idea behind this technique is to embed a nonconvex problem into a one-parameter family of convex optimization problems. Let us see how this technique can be applied to problem (10). First, let us rewrite the polynomials $h(\lambda)$ and $d(\lambda)$ as $h(\lambda) = \sum_{i=0}^{6} h_i(\lambda)$, $d(\lambda) = \sum_{i=0}^{6} d_i(\lambda)$, where $h_i(\lambda)$ and $d_i(\lambda)$ are homogeneous forms of degree i. Now, let us introduce the function $y(c; \lambda, \delta)$

$$y(c; \lambda, \delta) = \sum_{i=0}^{6} \frac{\delta^{6-i}}{c^{6-i}} [h_i(\lambda) - c\, d_i(\lambda)]. \tag{11}$$

We have the following properties:

1. for a fixed c, $y(c; \lambda, \delta)$ is a homogeneous form of degree 6 in λ and δ;
2. $y(\lambda, \delta) = y(c; \lambda, \delta)$ for all λ if $\delta = c$.

Hence, the form $y(c; \lambda, \delta)$ and the polynomial $y(\lambda, \delta)$ are equal on the plane $\delta = c$. In order to find J^*, let us observe that $\delta \geq 0$ because J^* is positive. Moreover, since $h(\lambda)$ is positive, then $y(\lambda, \delta) \geq 0$ for $\delta = 0$. This suggests that J^* can be computed as the minimum δ for which the function $y(\lambda, \delta)$ loses its positivity, that is

$$J^* = \min\{\delta : y(\lambda, \delta) \leq 0 \quad \text{for some} \ \ \lambda\}. \tag{12}$$

Using the homogeneous form $y(c; \lambda, \delta)$, (12) can be transformed into

$$J^* = \min\{c : y(c; \lambda, \delta) \leq 0 \quad \text{for some} \ \ \lambda, \delta\}. \tag{13}$$

The difference between (13) and (12) is the use of a homogeneous form, $y(c;\lambda,\delta)$, instead of a polynomial, $y(\lambda,\delta)$. Now, let us observe that $y(c;\lambda,\delta)$ can be written as

$$y(c;\lambda,\delta) = z^T(\lambda,\delta)Y(c)z(\lambda,\delta), \tag{14}$$

where $z(\lambda,\delta) \in \mathbb{R}^{10}$ is a base vector for the forms of degree 3 in the variables $\lambda_1, \lambda_2, \delta$, i.e.,

$$z(\lambda,\delta) = \left(\lambda_1^3 \;\; \lambda_1^2\lambda_2 \;\; \lambda_1^2\delta \;\; \lambda_1\lambda_2^2 \;\; \lambda_1\lambda_2\delta \;\; \lambda_1\delta^2 \;\; \lambda_2^3 \;\; \lambda_2^2\delta \;\; \lambda_2\delta^2 \;\; \delta^3\right)^T,$$

and $Y(c) \in \mathbb{R}^{10\times 10}$ is a symmetric matrix depending on c. Now, it is evident that positivity of the matrix $Y(c)$ ensures positivity of the homogeneous form $y(c;\lambda,\delta)$ (see (14)). Therefore, a lower bound c^* of J^* in (13) can be obtained by looking at the loss of positivity of $Y(c)$. To proceed, we observe that this matrix is not unique. In fact, for a given homogeneous form there is an infinite number of matrices that describe it, for the same vector $z(\lambda,\delta)$. So, we have to consider all these matrices in order to check the positivity of $y(c;\lambda,\delta)$. It is easy to show that all the symmetric matrices describing the form $y(c;\lambda,\delta)$ can be written as $Y(c) - L$, where $L \in \mathcal{L}$ and $\mathcal{L}$ is the linear set of symmetric matrices that describe the null form

$$\mathcal{L} = \left\{L = L^T \in \mathbb{R}^{10\times 10} : \;\; z^T(\lambda,\delta)Lz(\lambda,\delta) = 0 \;\; \forall \lambda,\delta\right\}.$$

Since $\mathcal{L}$ is a linear set, every element L can be parametrized linearly. Indeed, let $L(\alpha)$ be a generic element of $\mathcal{L}$. It can be shown that $\mathcal{L}$ has dimension 27 and, hence, $L(\alpha) = \sum_{i=1}^{27} \alpha_i L_i$ for a given base $L_1, L_2, \ldots, L_{27}$ of $\mathcal{L}$. Hence, all the symmetric matrices describing $y(c;\lambda,\delta)$ can be written as $Y(c) - L(\alpha)$. Summing up, a lower bound c^* of J^* can be obtained as

$$\begin{aligned} c^* &= \min_{c,\alpha} c \\ \text{subject to} \quad & \min_{\alpha} \lambda_M[L(\alpha) - Y(c)] > 0. \end{aligned} \tag{15}$$

This means that c^* can be computed via a sequence of convex optimizations indexed by the parameter c. Indeed, for a fixed c, the minimization of the maximum eigenvalue of a matrix parametrized linearly in its entries is a convex optimization problem that can be solved with standard LMI techniques [10], [1]. Moreover, a bisection algorithm on the scalar c can be employed to speed up the convergence.

It remains to discuss when the bound c^* is equal to the sought optimal J^*. It is obvious that this happens if and only if $y(c^*;\lambda,\delta)$ is positive semidefinite, i.e., there exists λ^* such that $y(c^*;\lambda^*,c^*) = 0$. In order to check this condition, a very simple test is proposed. Let $\mathcal{K}$ be defined as: $\mathcal{K} = \mathrm{Ker}\,[L(\alpha^*) - Y(c^*)]$, where α^* is the minimizing α for the constraint in (15). Then, $J^* = c^*$ if and only if there exists λ^* such that $z(\lambda^*,c^*) \in \mathcal{K}$. It is possible to show that, except for degenerate cases when $\dim(\mathcal{K}) > 1$, the last condition amounts to solving a very simple system in the unknown λ^*. In fact, when $\mathcal{K}$ is generated by one only vector k, λ^* can be obtained from the equation

$$z(\lambda^*,c^*) = \frac{c^{*3}}{k_{10}}k. \tag{16}$$

In order to solve the above equation, it is sufficient to observe that if (16) admits a solution λ^*, then

$$\lambda_1^* = c^*\frac{k_6}{k_{10}}, \quad \lambda_2^* = c^*\frac{k_9}{k_{10}}. \tag{17}$$

Now, we have just to check if λ^* given by (17) satisfies (16). If it does, then c^* is optimal and the fundamental matrix entries f^* solution of (5) are given by

$$f^* = f^*(\lambda^*) = v - ST(\lambda^*)P(\lambda^*)[T(\lambda^*)v - r(\lambda^*)]. \tag{18}$$

Criterion	Geometric error e_g
F_l	1.733
F_{cls}	0.6578
F_d	0.6560
F_d	0.6560

Fig. 1. Table 1: King's College sequence (40 used point correspondences) with epipolar lines superimposed.

Whenever c^* be not optimal, standard optimization procedures starting from the value of λ given by (17) can be employed for computing J^*. This is expected to prevent the achievement of local minima. However, in our experiments, we did not experience any case in which c^* is strictly less than J^*.

Remark. In order to avoid numerical problems due to too small values of c in (11), the above procedure can be implemented replacing δ in (9) by $\delta - 1$. This ensures $c \geq 1$.

5 Experimental Results

In this section, we present some results obtained by applying the proposed method for solving (4). The goal is to investigate its performance with respect to the linear criterion. Let us denote by e_a the algebraic error minimized in (4). In the sequel, we will refer to the estimate of the fundamental matrix given by the linear criterion with F_l; to the estimate provided by the proposed method, *constrained least-squares criterion*, with F_{cls}; and to the estimate provided by the distance to epipolar lines criterion with F_d when initialized by F_l, and with $\bar{F}_d$ when initialized by F_{cls}. In all cases, we scaled the image data in order to work with normalized data and set $w_i = 1$ (a different weighting is possible, but goes beyond the scope of the paper). The algorithm we use to compute F_{cls} is summarized below.

Algorithm for computing F_{cls}

1. Given the point correspondences u_i', u_i, form the polynomials $h(\lambda)$ and $d(\lambda)$ as shown, respectively, in (8) and (7).
2. Build a symmetric matrix function $Y(c)$ satisfying (14).
3. Solve the sequence of LMI problems (15).
4. Compute λ^* as shown in (17) and check for its optimality.
5. Retrieve f^* as shown in (18) and form F_{cls}.

In order to evaluate the algorithm performance, we define the geometric error e_g as the mean geometric distance between points and corresponding epipolar lines. Fig. 1 shows one of two typical views used to estimate the fundamental matrix. The point correspondences are found by a standard corner finder. The values of e_a obtained by F_l and F_{cls} are, respectively, 0.1551 and

Criterion	Geometric error e_g
F_l	1.255
F_{cls}	0.6852
F_d	0.5836
$\bar{F}_d$	0.5836

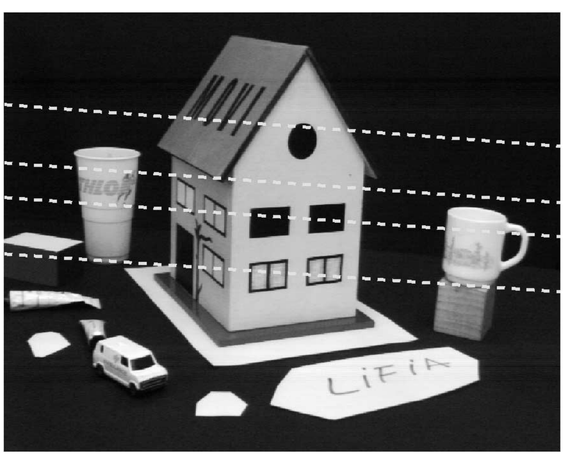
Fig. 2. Table 2: Cambridge street sequence (27 used point correspondences).

Criterion	Geometric error e_g
F_l	1.0891
F_{cls}	0.8852
F_d	0.7846
$\bar{F}_d$	0.7846

Fig. 4. Table 4: statue sequence (126 used point correspondences).

0.0272 (in order to compare the algebraic errors, we set the Frobenius norm of F_l and F_{cls} to 1, as the fundamental matrix is defined up to a scale factor). It can be seen that the proposed method outperforms the classic linear algorithm, as long as the algebraic error is considered. This has been observed in all experiments on real data.

Let us consider now the geometric error e_g. Table 1 reports the values of e_g given by linear and constrained least-squares criterion and by the distance to epipolar lines criterion initialized by F_l (F_d) and by F_{cls} ($\bar{F}_d$). As we can see, the geometric error achieved by F_{cls} is consistently smaller than the one achieved by F_l. Table 2 shows the geometric error obtained for the example in Fig. 2. Again, F_{cls} achieves a significant improvement with respect to F_l, while F_d and $\bar{F}_d$ coincide. Tables 3, 4, 5, and 6 report the geometric error obtained for the examples used in [7] and shown, respectively, in Figs. 3, 4, 5, and 6. Observe that, for the Oxford sequence and the calibration Jig one, not only F_{cls} achieves a smaller geometric error than F_l, but also $\bar{F}_d$ is smaller than F_d, indicating the presence of different local minima in the epipolar lines criterion. Moreover, in the calibration Jig example (Fig. 6), F_{cls} provides a better result even with respect to F_d. For the statue example (Fig. 4), Hartley's algebraic error minimization [7] has also been performed. It

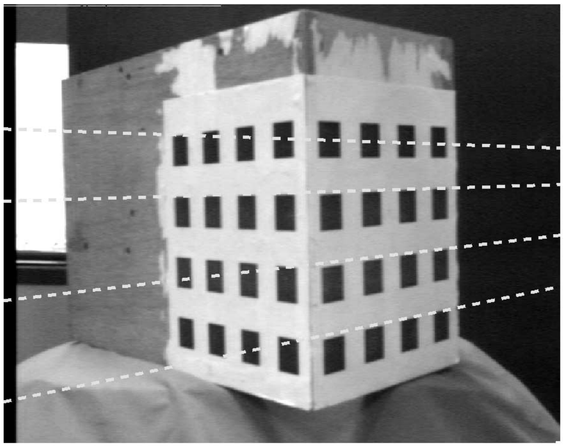


Criterion	Geometric error e_g
F_l	0.5409
F_{cls}	0.4690
F_d	0.3591
$\bar{F}_d$	0.3591

Fig. 3. Table 3: house sequence (26 used point correspondences).

Criterion	Geometric error e_g
F_l	0.4503
F_{cls}	0.4406
F_d	0.1791
$\bar{F}_d$	0.1607

Fig. 5. Table 5: Oxford basement sequence (100 used point correspondences).



Criterion	Geometric error e_g
F_l	0.4066
F_{cls}	0.1844
F_d	0.1943
F_d	0.1844

Fig. 6. Table 6: calibration Jig sequence (128 used accurate point correspondences).

achieves $e_a = 4.59 \times 10^{-4}$ (which leads to $e_g = 0.8852$ when the geometric error is minimized) if initialized by the epipoles provided by F_l, but $e_a = 0.0462$ ($e_g = 24.49$) if initialized by $(0, 0)$. This clearly shows the presence of local minima in Hartley's minimization of the algebraic error and, hence, the dependence of the found solution on the chosen starting point. As we can see from the above results, the solution provided by the proposed method returns smaller algebraic and geometric errors with respect to the linear criterion. Moreover, initializing nonlinear criteria with the obtained solution allows us to achieve more accurate estimates of the fundamental matrix.

6 Conclusions

In this paper, we have proposed a new method for the estimation of the fundamental matrix. It consists of minimizing the same algebraic error as that used in the linear criterion, but taking into account explicitly the rank constraint. It has been shown how the resulting constrained least-squares problem can be solved using recently developed convexification techniques. The experiments show that this method provides a more accurate estimate of the fundamental matrix compared to that given by the linear criterion in terms of epipolar geometry. This suggests that our estimation procedure can be used to initialize more complex nonconvex criteria minimizing the geometric distance in order to obtain better results.

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