A Differential approach to Shape from Polarisation

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Spherical Coordinates Recap

\[ n(x) = (\cos \theta \sin \phi, \sin \theta \sin \phi, \cos \phi) \]

Differential surface form:

\[ \cos \theta = k z_x \]
\[ \sin \theta = k z_y \]

\[ \tan \theta = \frac{z_y}{z_x} \]

Same constant \( k \)
Shape from Polarisation

Target object-dielectric

rotating polariser

RGB camera

\[ I(\theta_{pol}) = I_+ + I_- \cos(2\theta_{pol} - 2\theta) \]
Background

\[ \rho = \frac{I_-}{I_+} \] is a function of \( \phi \)

Diffuse:

\[ \rho = \frac{\left( \mu - \frac{1}{\mu} \right)^2 \sin^2 \phi}{2 + 2\mu^2 - \left( \mu + \frac{1}{\mu} \right)^2 \sin^2 \phi + 4 \cos \phi \sqrt{\mu^2 - \sin^2 \phi}} \]

Specular:

\[ \rho = \frac{2 \sin^2 \phi \cos \phi \sqrt{\mu^2 - \sin^2 \phi}}{\mu^2 - \sin^2 \phi - \mu \sin^2 \phi + 2 \sin^4 \phi} \]

Contribution:

- the proposed differential model is reflection type (diffuse/specular) independent
- iso-contours of the surface are directly provided from the characteristic field of the model
- the SfP model is easy to merge with other techniques having suitable differential formulation

Easy to find the sinusoid but:

- Two-fold ambiguity on \( \theta \)
- Solve for \( \phi \)
Differential Formulation

Applying polarisation equation to $0$ and $\frac{\pi}{4}$ angles:

\[ I_0 = I_+ + I_- (2 \cos \theta^2 - 1) \]
\[ I_{\frac{\pi}{4}} = I_+ + I_- (2 \sin \theta \cos \theta) \]

\[ \frac{I_0 - I_+ + I_-}{2I_-} = \cos^2 \theta \]
\[ \frac{I_{\frac{\pi}{4}} - I_+}{2I_-} = \sin \theta \cos \theta \]

\[ \tan \theta = \frac{I_0 - I_+ + I_-}{I_{\frac{\pi}{4}} - I_+} \]
\[ z_x \left( -I_{\frac{\pi}{4}} + I_+ \right) + z_y \left( I_0 - I_+ + I_- \right) = 0 \]
Real Data Experiments

Painted porcelain cup  Mannequin head¹  Plastic ball and a cement statue

Merging with Photometric Stereo

Polarisation equation:
\[ b_{pol}(x) \cdot \nabla z(x) = 0 \]

Photometric stereo\(^2\):
\[ b_{ps}(x, z) \cdot \nabla z(x) = s_{ps}(x, z) \]

Combine:
\[ \min_z \left\| \begin{bmatrix} b_{ps} \\ b_{pol} \end{bmatrix} \cdot \nabla z - \begin{bmatrix} s_{ps} \\ 0 \end{bmatrix} \right\|_{L^2} + \lambda \left\| z - z_0 \right\|_{L^2} \]

Well defined problem even with 2 lights photometric stereo

Real Data Experiment

Come to our poster for more details!