6. 2001 PAPER 4

(a) Convolution is a technique for finding the output of a linear system ($y(t)$), given the input $x(t)$, and the IMPULSE RESPONSE $g(t)$, of the system.

$$y(t) = \int_{-\infty}^{t} x(\tau)\, g(t-\tau)\, d\tau .$$

The formula is derived using the principle of SUPERPOSITION, and considering the input as being composed of a sum of impulses. The output can thus be considered as a sum of impulse responses scaled by the input giving rise to each of them. In the limit this ~~sum~~ sum becomes the convolution integral above.

(b) Using the formula above to get the step response

$$y(t) = \int_{-\infty}^{t} H(\tau)\, g(t-\tau)\, d\tau = \int_{0}^{t} g(t-\tau)\, d\tau$$

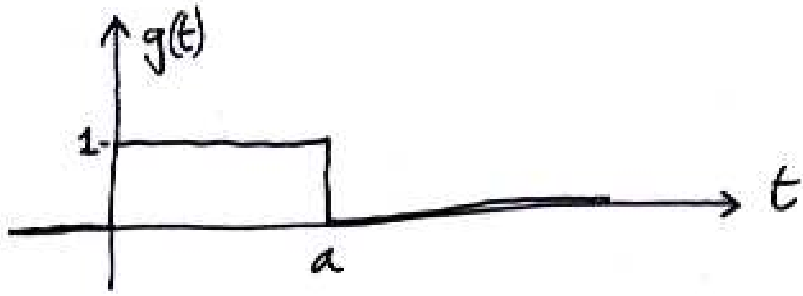
6 contd

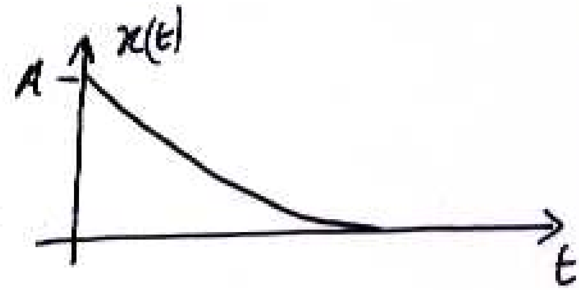
let $u = t - \tau \Rightarrow du = -d\tau$ ②

$$\Rightarrow y(t) = -\int_t^0 g(u)\,du = \int_0^t g(u)\,du$$

which I can write as $y(t) = \int_0^t g(\tau)\,d\tau$

(c)





(i)

Both $g(t)$ and $x(t)$ are 0 for $t < 0$ so use

$$y(t) = \int_0^t g(\tau)\,x(t-\tau)\,d\tau = \int_0^t A e^{-b(t-\tau)}\,d\tau \quad , t < a$$

$$= \int_0^a A e^{-b(t-\tau)}\,d\tau \quad , t > a$$

So for $t < a$

$$\int_0^t A e^{-b(t-\tau)} d\tau = \left[\frac{A}{b} e^{-b(t-\tau)}\right]_0^t = \frac{A}{b}\left(1 - e^{-bt}\right)$$

and for $t > a$

$$\int_0^a A e^{-b(t-\tau)} d\tau = \left[\frac{A}{b} e^{-b(t-\tau)}\right]_0^a = \frac{A}{b}\left(e^{-b(t-a)} - e^{-bt}\right)$$

$$= \frac{A}{b}\left(e^{ba} - 1\right) e^{-bt}$$

(ii)

Thus