

9.(a)

1997 PAPER 4

①

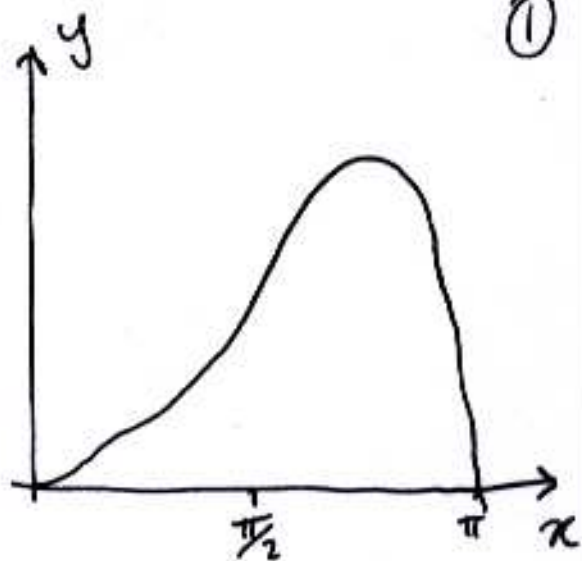
$$y = \pi \sin(\pi x)$$

$$y' = \sin(\pi x) + \pi \cos(\pi x)$$

$$x = 0 \Rightarrow y = 0 \text{ and } y' = 0$$

$$x = \frac{\pi}{2} \Rightarrow y = \frac{\pi}{2} \text{ and } y' = 1$$

$$x = \pi \Rightarrow y = 0 \text{ and } y' = -\pi$$



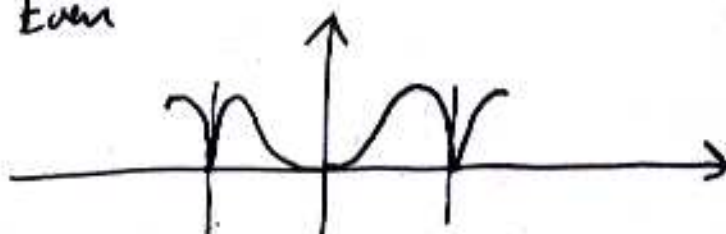
(b) Odd



continuous gradient.

Converges faster.

Even



discontinuous gradient

Entirely ODD with zero mean $\Rightarrow a_n = d = 0$.

We just need to find b_n .

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} y \sin n x \, dx = \frac{2}{\pi} \int_0^{\pi} \pi \sin \pi x \sin n x \, dx$$

$$= \frac{1}{\pi} \int_0^{\pi} \pi [\cos(\pi - n\pi) - \cos(\pi + n\pi)] \, dx$$

9 contd.)

So:

$$b_n = \frac{1}{\pi} \left[\left[x \frac{\sin(x-nx)}{1-n} \right]_0^\pi - \int_0^\pi \frac{\sin(x-nx)}{1-n} dx \right]$$

↗ Provided $n \neq 1$

$$- \left[x \frac{\sin(x+nx)}{1+n} \right]_0^\pi + \int_0^\pi \frac{\sin(x+nx)}{1+n} dx$$

$$= \frac{1}{\pi} \left[\frac{\pi \sin([1-n]\pi)}{1-n} - \left[-\frac{\cos(x-nx)}{(1-n)^2} - \frac{\pi \sin([1+n]\pi)}{1+n} \right]_0^\pi + \left[-\frac{\cos(x+nx)}{(1+n)^2} \right]_0^\pi \right]$$

$$= \frac{1}{\pi} \left[\frac{\cos([1-n]\pi) - 1}{(1-n)^2} + \frac{1 - \cos([1+n]\pi)}{(1+n)^2} \right]$$

↑
zero when $(n-1)$ even
ie when n ODD

↑
zero when $(1+n)$ even
ie when n ODD.

⇒ $b_n = 0$ for n ODD provided $n > 1$, deal with $n=1$ later.

$$b_n = \frac{1}{\pi} \left[\frac{-2}{(1-n)^2} + \frac{2}{(1+n)^2} \right] \text{ for } n \text{ EVEN.}$$

d contd.)

$$\text{For } n \text{ even } b_n = \frac{1}{\pi} \left(\frac{-2(1+n)^2 + 2(1-n)^2}{(1-n)^2 (1+n)^2} \right)$$

$$= \frac{2}{\pi} \left(\frac{1-2n+n^2 - 1-2n-n^2}{(1-n^2)^2} \right)$$

$$= \frac{2}{\pi} \left(\frac{-4n}{(1-n^2)^2} \right) = \frac{-8n}{\pi (n^2-1)^2} \quad \text{for } n \text{ even}$$

What about $n=1$? In this case we have.

$$b_{1n} = \frac{1}{\pi} \int_0^{\pi} x [\cos(x-nx) - \cos(x+nx)] dx$$

$$= \frac{1}{\pi} \int_0^{\pi} x - x \cos(2x) dx$$

SPECIAL
CASE.

$$= \frac{1}{\pi} \left[\left[\frac{1}{2} x^2 \right]_0^{\pi} - \left[x \frac{\sin 2x}{2} \right]_0^{\pi} + \int_0^{\pi} \frac{\sin 2x}{2} dx \right]$$

9 contd / So, for b_n when $n=1$ we have

(4)

$$b_1 = \frac{1}{\pi} \left[\frac{\pi^2}{2} + \left[\frac{-\cos 2x}{4} \right]_0^{\pi} \right] = \frac{1}{\pi} \left[\frac{\pi^2}{2} + 0 \right] = \frac{\pi}{2}$$

$$\text{So } b_1 = \frac{\pi}{2}$$

$$b_2 = \frac{-16}{9\pi}$$

$$b_3 = 0$$

$$b_4 = \frac{-32}{225\pi}$$

Thus

$$y \approx \frac{\pi}{2} \sin x - \frac{16}{9\pi} \sin 2x - \frac{32}{225\pi} \sin 4x - \dots$$