1997 PAPER 4

9.(a)

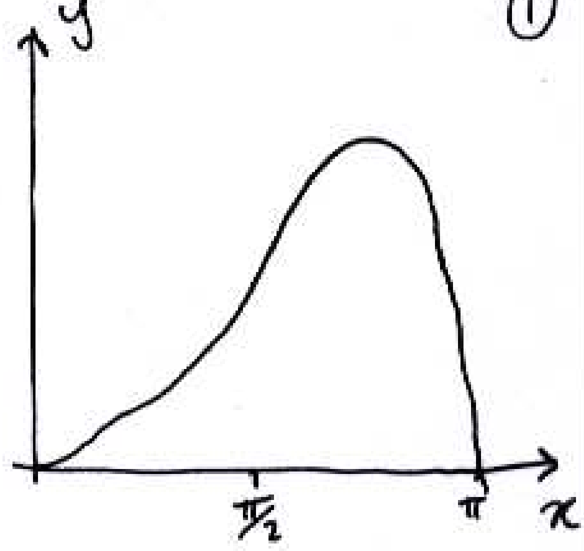
$$y = x\sin(x)$$

$$y' = \sin(x) + x\cos(x)$$

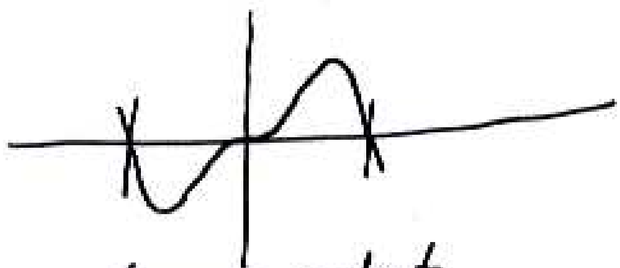
$x = 0 \Rightarrow y = 0$ and $y' = 0$

$x = \frac{\pi}{2} \Rightarrow y = \frac{\pi}{2}$ and $y' = 1$

$x = \pi \Rightarrow y = 0$ and $y' = -\pi$



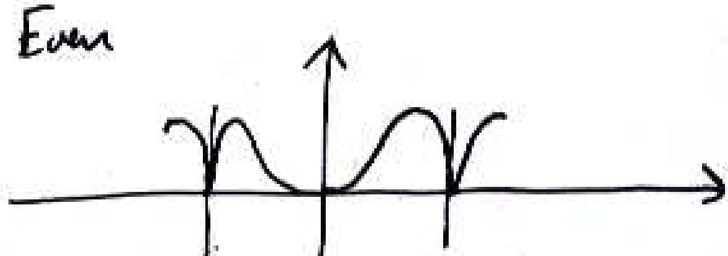
(b) Odd



continuous gradient.

<u>Converges faster.</u>

Even



discontinuous gradient

Entirely ODD with zero mean $\Rightarrow a_n = d = 0$.

We just need to find b_n.

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} y\sin nx\,dx = \frac{2}{\pi}\int_0^{\pi} x\sin x\sin nx\,dx$$

$$= \frac{1}{\pi}\int_0^{\pi} x\left[\cos(x-nx) - \cos(x+nx)\right]dx$$

9 contd.) ②

So:

$$b_n = \frac{1}{\pi}\left[\left[x\,\frac{\sin(x-nx)}{1-n} \right]_0^{\pi} - \int_0^{\pi} \frac{\sin(x-nx)}{1-n}\,dx \right.$$

Provided $n \neq 1$

$$\left. - \left[x\,\frac{\sin(x+nx)}{1+n} \right]_0^{\pi} + \int_0^{\pi} \frac{\sin(x+nx)}{1+n}\,dx \right]$$

$$= \frac{1}{\pi}\left[\cancel{\pi\frac{\sin([1-n]\pi)}{1-n}} - \left[\frac{-\cos(x-nx)}{(1-n)^2} \right]_0^{\pi} - \cancel{\frac{\pi\sin([1+n]\pi)}{1+n}} + \left[\frac{-\cos(x+nx)}{(1+n)^2} \right]_0^{\pi} \right]$$

$$= \frac{1}{\pi}\left[\frac{\cos([1-n]\pi) - 1}{(1-n)^2} + \frac{1-\cos([1+n]\pi)}{(1+n)^2} \right]$$

zero when $(n-1)$ even ie when n ODD

zero when $(1+n)$ even ie when n ODD.

$\Rightarrow b_n = 0$ for n ODD provided $n > 1$, deal with $n=1$ later.

$$b_n = \frac{1}{\pi}\left[\frac{-2}{(1-n)^2} + \frac{2}{(1+n)^2} \right] \text{ for } n \text{ EVEN.}$$

9 contd.

For n even $\quad b_n = \dfrac{1}{\pi}\left(\dfrac{-2(1+n)^2 + 2(1-n)^2}{(1-n)^2(1+n)^2} \right)$

$$= \frac{2}{\pi}\left(\frac{1-2n+n^2-1-2n-n^2}{(1-n^2)^2} \right)$$

$$= \frac{2}{\pi}\left(\frac{-4n}{(1-n^2)^2} \right) = \frac{-8n}{\pi(n^2-1)^2} \quad \text{for } n \text{ even}$$

What about $n=1$? In this case we have.

$$b_1 = \frac{1}{\pi}\int_0^{\pi} x\left[\cos(x-nx) - \cos(x+nx)\right] dx$$

SPECIAL CASE.

$$= \frac{1}{\pi}\int_0^{\pi} x - x\cos(2x)\, dx$$

$$= \frac{1}{\pi}\left[\left[\tfrac{1}{2}x^2\right]_0^{\pi} - \left[x\frac{\sin 2x}{2}\right]_0^{\pi} + \int_0^{\pi} \frac{\sin 2x}{2}\, dx \right]$$

9 contd/ so, for b_n when $n=1$ we have

$$b_1 = \frac{1}{\pi}\left[\frac{\pi^2}{2} + \left[\frac{-\cos 2x}{4}\right]_0^{\pi}\right] = \frac{1}{\pi}\left[\frac{\pi^2}{2} + 0\right] = \frac{\pi}{2}$$

So $b_1 = \frac{\pi}{2}$

$b_2 = \frac{-16}{9\pi}$

$b_3 = 0$

$b_4 = \frac{-32}{225\pi}$

Thus

$$y \approx \frac{\pi}{2}\sin x - \frac{16}{9\pi}\sin 2x - \frac{32}{225\pi}\sin 4x - \cdots\cdots$$