Question

If $\frac{dy}{dt} + 5t = H(t)$

explain why $y(t)$ is not infinite at $t=0$.

Conceptual explanation

If y is of the form $A\,\delta(t)$ 

then $\dot{y}$ is of the form 

and you cannot get a step function by adding multiples of these functions together.

Algebraic explanation

Let $y(t) = A\,\delta(t)$

$$\Rightarrow \frac{dy}{dt} = -\frac{A}{t}\,\delta(t)$$

(See Wolfram Mathworld)

$$\Rightarrow \frac{dy}{dt} + 5y = \frac{-A}{t}\,\delta(t) + 5A\,\delta(t)$$

Integrate term-by-term from $(0-\epsilon)$ to $(0+\epsilon)$

$$I_{LHS} = 0 + 5A$$

Integrate $H(t)$ from $(0-\epsilon)$ to $(0+\epsilon)$

$$I_{RHS} = \epsilon$$

Hence I_{LHS} cannot equal I_{RHS} as one is a function of ϵ and the other is not.

Hence the equation cannot be satisfied by a solution of this form.