

# Linear Transforms in Automatic Speech Recognition: Estimation Procedures & Integration of Diverse Acoustic Data

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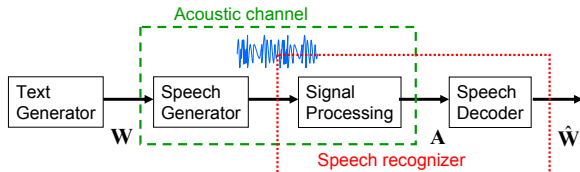
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# Introduction



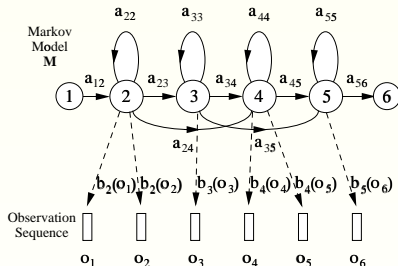
Source-channel model of speech recognition

- ▶ Purpose of speech recognition is to find an estimate  $\hat{W}$  of  $W$
- ▶ *Maximum a posteriori* (MAP) decoder:

$$\hat{W} = \underset{W'}{\operatorname{argmax}} P(W'|A) = \underset{W'}{\operatorname{argmax}} P(W')P(A|W')$$

- ▶ To compute  $P(W)$  we use a statistical *language model*  $\tilde{P}(W)$
- ▶ To compute  $P(A|W)$  we use a statistical *acoustic model*  $P_{\theta}(A|W)$
- ▶ We will investigate the use of linear transforms in acoustic models to
  - ▶ improve acoustic modeling
  - ▶ work with diverse acoustic data

## Acoustic Model



- ▶ The Hidden Markov Model (HMM) is the standard acoustic model
- ▶ Output emission density is modeled as a Gaussian :  $b_s(o_T; \theta) = \mathcal{N}(o_T; \mu_s, \Sigma_s)$
- ▶ Applying affine transform  $T = [b \ A]$  to the observation vector  $\zeta = [1 \ o']'$  ( $Ao + b = T\zeta$ ) the Gaussian density is reparameterized as:

$$b_s(\zeta; \theta) = |A| \cdot \mathcal{N}(T\zeta; \mu_s, \Sigma_s) = \frac{|A|}{\sqrt{(2\pi)^m |\Sigma_s|}} e^{-\frac{1}{2}(T\zeta - \mu_s)' \Sigma_s^{-1} (T\zeta - \mu_s)}$$

## Motivation

- ▶ Most ASR systems
  - ▶ HMM-based acoustic models
  - ▶ Gaussian state-dependent emission densities
    - Diagonal covariance assumption
  - ▶ Trained from homogeneous data collections similar to the recognition task
  - ▶ Parameters estimated via Maximum Likelihood or discriminative criteria
- ▶ Linear transforms in ASR
  - ▶ Feature-space :
    - Maximum Likelihood Linear Transform (MLLT)
    - Heteroscedastic Linear Discriminant Analysis (HLDA)
  - ▶ Model-space :
    - Maximum Likelihood Linear Regression (MLLR)

## Goals

- ▶ Discriminative estimation of feature-based transforms
  - ▶ Estimate transforms under discriminative criteria rather than ML
  - ▶ Incorporate discriminative transforms in discriminative HMM training
- ▶ Maximum-A-Posteriori (MAP) estimation of feature-based transforms
  - ▶ Estimate transforms under MAP criterion
  - ▶ Useful for dealing with problems posed by sparse training data
- ▶ Develop acoustic normalization procedures for the integration of diverse acoustic data
  - ▶ Enlarge acoustic training set with speech data that would otherwise lead to performance degradation

# Outline

## Discriminative Estimation Procedures

- Discriminative Likelihood Linear Transforms
- DLLT Performance in Large Vocabulary Speech Recognition
- DLLT Summary

## Maximum-A-Posteriori Estimation Procedures

## Integration of Diverse Acoustic Data

- Cross-Corpus Acoustic Normalization
- Acoustic Corpora Description
- Experimental Results for Cross-Corpus Normalization
- Cross-Corpus Normalization Summary

## Highlights of Thesis Contributions

## Future Work

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# Discriminative Estimation of Linear Transforms

- ▶ Focus on MLLT technique:
  - ▶ Transforms acoustic data to ease diagonal covariance assumption
  - ▶ Estimation of transforms via ML criterion
- ▶ Discriminative training of HMMs useful in LVCSR tasks
- ▶ Success triggered an interest in the use of discriminative linear transforms
  - ▶ *Maximum Mutual Information Linear Regression*
  - ▶ *Conditional Maximum Likelihood Linear Regression*
  - ▶ Both techniques use model-space linear transforms
- ▶ **Prior Work:** MLLT was developed as ML technique, but has also been used with MMI
  - ▶ The AT&T LVCSR-2001 system used:
    - ▶ feature-based transforms obtained by ML estimation techniques, and
    - ▶ were then fixed throughout the subsequent iterations of MMI model estimation
- ▶ **Goal:** Estimation of feature-based transforms under the Conditional Maximum Likelihood (CML) criterion

## Conditional Maximum Likelihood Auxiliary Function

- ▶ CML criterion uses a general auxiliary function similar to EM

$$\bar{\theta} : \sum_s \sum_{\tau} (\gamma_{s,\tau}(\theta) - \gamma_{s,\tau}^g(\theta)) \cdot \nabla_{\theta} \log q(o_{\tau}|s; \bar{\theta}) \\ + \sum_s D_s \int q(o|s; \theta) \nabla_{\theta} \log q(o|s; \bar{\theta}) do = 0$$

- ▶ State dependent distributions are reparametrized to incorporate linear transforms
- ▶ CML version of MLLT is readily obtained
- ▶ Goal is to maximize  $P(W|A; \theta)$  by alternatively updating the transforms and HMM parameters
- ▶ As a result both transforms and HMM parameters are estimated discriminatively

## Discriminative Likelihood Linear Transforms

- ▶ Apply affine transform to the observation vector :  $T\zeta = Ao + b$
- ▶ Under the preceding model, emission density of state  $s$  becomes

$$q(\zeta|s; \theta) = \frac{|A_{\mathcal{R}(s)}|}{\sqrt{(2\pi)^m |\Sigma_s|}} e^{-\frac{1}{2}(T_{\mathcal{R}(s)}\zeta - \mu_s)^T \Sigma_s^{-1} (T_{\mathcal{R}(s)}\zeta - \mu_s)}$$

**Objective:** We estimate transforms & HMM parameters under the CML criterion

- ▶ The transforms obtained under this criterion are termed

Discriminative Likelihood Linear Transforms (DLLT)

- ▶ This estimation is performed as a two-stage iterative procedure:
  - a) Maximize CML criterion with respect to the transforms while keeping the Gaussian parameters fixed
  - b) Compute the Gaussian parameters using the updated values of the transforms

## DLLT Estimation

As in MLLT (Gales '97), the  $i^{\text{th}}$  row of the transformation matrix is found by

$$[\bar{T}_r]_i = (\alpha p_i + k_{r,i}) G_{r,i}^{-1}$$

where

$$G_{r,i} = \sum_{s: \mathcal{R}(s)=r} \frac{1}{\sigma_{s,i}^2} \left( \sum_{\tau} (\gamma_{s,\tau}(\theta) - \gamma_{s,\tau}^g(\theta)) \hat{\zeta}_{\tau} \hat{\zeta}_{\tau}^T + D_s J_s \right)$$

$$k_{r,i} = \sum_{s: \mathcal{R}(s)=r} \frac{\mu_{s,i}}{\sigma_{s,i}^2} \left( \sum_{\tau} (\gamma_{s,\tau}(\theta) - \gamma_{s,\tau}^g(\theta)) \hat{\zeta}_{\tau}^T + D_s [J_s]_1 \right)$$

$$J_s = \begin{bmatrix} 1 & (A_r^{-1}(\mu_s - b_r))^T \\ A_r^{-1}(\mu_s - b_r) & A_r^{-1}(\Sigma_s + (\mu_s - b_r)(\mu_s - b_r)^T)A_r^{-1T} \end{bmatrix}.$$

## DLLT: System Description

**Task:** English conversational telephone speech transcription

- ▶ Acoustic Training/Test Set
  - ▶ Training: 16.4 Hrs Switchboard-1 & 0.5 Hr CallHome
  - ▶ Test: 866 utts 2000 Switchboard-1 evaluation (SWBD1) & 913 utts 1998 Switchboard-2 evaluation (SWBD2)
  - ▶ 2001 JHU LVCSR minitrain/minitest system
- ▶ Acoustic Model
  - ▶ Standard HTK flat-start training procedure
  - ▶ Tied state, cross-word, context-dependent triphones
  - ▶ 4000 unique triphone states
  - ▶ 6 mixtures per speech state
- ▶ Language Model
  - ▶ 33k-word trigram language model
- ▶ Fixed number (450) of linear transforms

## DLLT Results: ML vs. CML transforms

**Goal:** Compare ML trained transforms to CML trained transforms

- ▶ Gaussian parameters fixed throughout transform updates
- ▶ Test whether CML transforms improve over ML transforms
- ▶ Validate CML as a modeling procedure

Transform reestimation only		
	Word Error Rate (%)	
	SWBD1	SWBD2
ML	41.1	51.1
ML+MLLT	39.1	50.3
ML+DLLT	38.5	49.7

- ▶ CML transforms outperform ML transforms

## DLLT Results: Fully discriminative training

Goal: Investigate fully discriminative training compared to ML training

A			B		
DLLT			MLLT and DLLT		
	Word Error Rate (%)			Word Error Rate (%)	
	SWBD1	SWBD2		SWBD1	SWBD2
ML	41.1	51.1	ML	41.1	51.1
DLLT-1it	38.2	49.2	MLLT-1it	38.4	49.6
DLLT-2it	37.3	48.9	MLLT-2it	38.2	49.5
			MLLT-6it	37.8	49.0
			MLLT+DLLT-1it	37.1	48.4

- ▶ DLLT outperforms MLLT
- ▶ DLLT is additive with MLLT
- ▶ DLLT works best when initialized by MLLT

# DLLT Summary

- ▶ Developed discriminative estimation procedures for feature-based transforms
- ▶ Integrated discriminative linear transforms into CML estimation for LVCSR
- ▶ Discriminative versions outperform ML training
- ▶ Validated CML as a modeling procedure
- ▶ DLLT used in the JHU LVCSR-2002 Evaluation system with similar success
- ▶ Discriminative linear transforms became standard in state-of-the-art ASR systems

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## Maximum-A-Posteriori Estimation of Linear Transforms

- ▶ Provides a way of incorporating prior information about the transformation parameters in the training process
- ▶ MAP estimate of transform  $T$ :

$$\bar{T} = \arg \max_T p(\hat{\delta}_1^j | M_{\hat{w}_1^{\hat{n}}}, T; \theta) p(T)$$

- ▶ MAP estimate can be formulated in an EM algorithm

$$R(\bar{T}, T) = Q(\bar{T}, T) + \log p(\bar{T})$$

where  $Q(\bar{T}, T)$  is the conventional ML auxiliary function

## Selection of the prior density

- ▶ MAP solution is strongly related to
  - a) the choice of the prior density and
  - b) the specification of the parameters for the prior densities
- ▶ Use the matrix variate normal distribution:

$$p(T; \eta) \propto |\Phi|^{-(m+1)/2} |\Psi|^{-m/2} \exp\left\{-\frac{1}{2} \text{tr}(\Phi^{-1}(T - \Lambda)\Psi^{-1}(T - \Lambda)^T)\right\}$$

where  $T, \Lambda \in \mathbb{R}^{m \times (m+1)}$ ,  $\Phi \in \mathbb{R}^{m \times m}$ ,  $\Phi > 0$ ,  $\Psi \in \mathbb{R}^{(m+1) \times (m+1)}$ ,  $\Psi > 0$ .

1. Closed-form solution for the transformation parameters
2. Related to the location-scale family of distributions
  - ▶  $\Lambda$  is the location parameter and  $\Psi$ ,  $\Phi$  are the scale parameters

## MAP Feature-Space Transforms

As in MLLT and DLLT, the  $i^{\text{th}}$  row of the transformation matrix is found by

$$[\bar{T}]_i = (\tilde{\alpha} p_i + \tilde{k}_i) \tilde{G}_i^{-1}$$

where

$$\tilde{G}_i = G_i + \frac{1}{2}(\phi^{-1})_{i,i}(\Psi^{-1} + \Psi^{-T})$$

and

$$\begin{aligned} \tilde{k}_i &= k_i + \frac{1}{2}[\Phi^{-1} \wedge \Psi^{-1}]_i + \frac{1}{2}[\Phi^{-T} \wedge \Psi^{-T}]_i \\ &\quad - \frac{1}{2} \sum_{j \neq i} [T]_j \left( (\phi^{-1})_{i,j} \Psi^{-1} + (\phi^{-1})_{j,i} \Psi^{-T} \right) \end{aligned}$$

- ▶ The MAP and ML update equations are similar, except for the additional terms in the definition of  $\tilde{G}_i$ ,  $\tilde{k}_i$  and  $\tilde{\alpha}$ .

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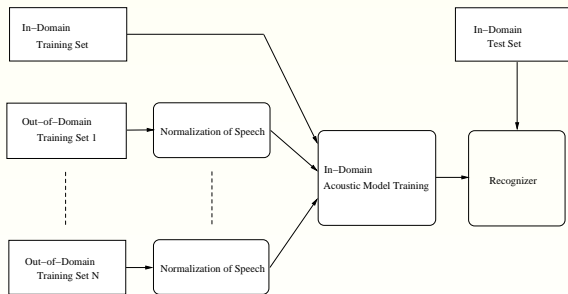
# Motivation

- ▶ State-of-the-art ASR systems require the collection of huge amounts of transcribed speech for the task which the ASR system will be applied to
- ▶ Collection and transcription of speech is expensive and time-consuming
- ▶ Available speech collections come from a variety of sources
- ▶ **Great challenge:** Integration of heterogeneous data sources in acoustic training

# Integration of Diverse Acoustic Data

- ▶ Speech collections differ due to a wide range of factors:
  - ▶ Acoustic channel, sampling rate
  - ▶ Speaking style, speaker age and education
  - ▶ Domain or topic, dialect, language
- ▶ Incorporating diverse acoustic data may lead to performance degradation
- ▶ **Objective:** Develop acoustic normalization techniques that:
  - ▶ Augment acoustic training set with out-of-domain data

## Acoustic Normalization Overview



- ▶ Map the out-of-domain feature space onto the in-domain data
- ▶ Employ linear transforms
- ▶ Obtain a larger in-domain training set by transforming the out-of-domain data
- ▶ Performance measured by improvements on the in-domain test set

## Cross-Corpus Acoustic Normalization

- ▶ Collection of  $C$  training sets, indexed by  $c$ 
  - ▶  $c = t$  denotes the in-domain data set
- ▶ Assume in-domain training data sufficient representative of in-domain test data
- ▶ Transforms estimated over the out-of-domain training sets
- ▶ Emission density of state  $s$  is reparameterized as

$$q(\zeta | s, c; \theta) = \frac{|A^{(c)}|}{\sqrt{(2\pi)^m |\Sigma_s|}} e^{-\frac{1}{2}(T^{(c)}\zeta - \mu_s)' \Sigma_s^{-1} (T^{(c)}\zeta - \mu_s)}$$

- ▶ Distribution depends on  $c$ ; the training set to which is applied
- ▶ Assume in-domain data does not need to be normalized at the corpus level :

$$A^{(t)} = \mathbf{I} \text{ and } b^{(t)} = \mathbf{0}$$

## Cross-Corpus Parameter Estimation with Normalization

- ▶ Incorporate source identity into the observed random process

$$(\hat{w}_1^{\hat{n}}, \hat{o}_1^{\hat{l}}, \hat{c}_1^{\hat{l}})$$

- ▶ Goal: Estimate transforms and HMM parameters via ML

$$\theta^* = \operatorname{argmax}_{\theta} p(\hat{o}_1^{\hat{l}} | M_{\hat{w}_1^{\hat{n}}}, \hat{c}_1^{\hat{l}}; \theta)$$

- ▶ This estimation is performed as a two-stage iterative procedure:
  - ▶ Maximize ML criterion with respect to the transforms while keeping the Gaussian parameters fixed
  - ▶ Compute the Gaussian parameters using the updated values of the transforms

## Acoustic Corpora Description

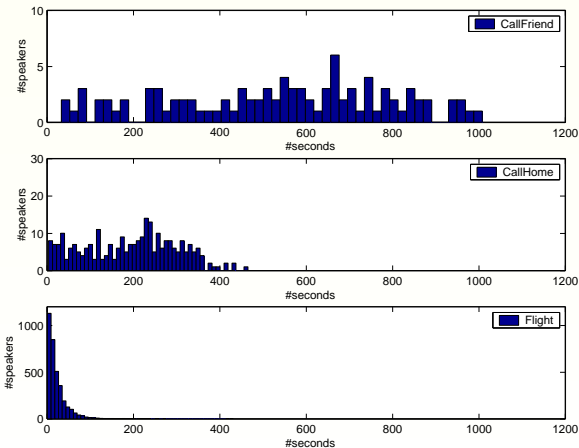
**Task:** Mandarin conversational telephone speech transcription for the CallFriend domain

**Available acoustic corpora:** Three conversational Mandarin databases collected over the telephone

	Data Sources in Training	#Hours	#Conversations
In-domain	CallFriend (CF)	14	42
Out-of-domain	CallHome (CH)	14	100
	Flight (FL)	22	1800

- ▶ CallFriend : both parties located in USA and Canada
- ▶ CallHome : calls originated in N. America but placed overseas
- ▶ Flight : based entirely in China
- ▶ Speakers in CallFriend & CallHome took advantage of a free phone call
- ▶ Flight : conversations between travel agents and customers about flight enquiries and reservations

# Acoustic Corpora Analysis



► Majority of the speakers in the Flight corpus have limited training data!

## System Description

- ▶ Baseline Acoustic Models
  - ▶ Standard HTK flat-start training procedure
  - ▶ Number of unique states depends on the amount of training data
- ▶ Language Model (LM)
  - ▶ Three bigram LMs trained from each set of transcriptions
  - ▶ Linearly interpolated with weight chosen to minimize perplexity on held-out CallFriend transcriptions

Corpus	CF	CH	FL
Weight	0.77	0.21	0.02

- ▶ This bigram used for *all* decoding experiments

## Unnormalized Out-of-Domain Acoustic Data

Data Sources in Training				Character Error Rate (%)	
CF	CH	FL	Total Hours	SI	SI+MLLR
✓			14	60.8	58.7
	✓		14	62.2	59.8
		✓	22	69.2	65.8
✓	✓		28	57.9	55.9
✓		✓	36	60.8	58.7
✓	✓	✓	50	59.3	56.6

- ▶ CallHome comparable to CallFriend
- ▶ Flight significantly worse than CallFriend and CallHome
- ▶ Adding Flight to CallFriend & CallHome degrades performance !

## Normalized Out-of-Domain Acoustic Data

Data Sources & Normalization				Character Error Rate (%)	
CF	CH	FL	#transforms	SI	SI+MLLR
I	I			57.9	55.9
I	T		1 per corpus	57.6	55.8
I	I	I		59.3	56.6
I	I	T	1 per corpus	58.1	55.7
I	T	T	1 per corpus	57.8	55.5

- ▶ 'T' / 'I' indicates that a source was included in training with / without normalization, resp.
- ▶ 1 (global) transform per corpus
- ▶ Cross-corpus normalization prevents degradation in performance

## Speaker-to-Corpus Normalized Out-of-Domain Acoustic Data

Data Sources & Normalization				Estimation Criterion	CER	
CF	CH	FL	#transforms		SI	SI+MLLR
I	T	T	1 per corpus	ML	57.8	55.5
I	T	T	1 per speaker	ML	57.7	55.2
I	T	T	1 per speaker	MAP	57.6	54.9

- ▶ A significant amount of speakers in the out-of-domain corpora have limited amounts of training data.
- ▶ MAP estimation procedure reduced the overfitting of the speaker-level transforms.

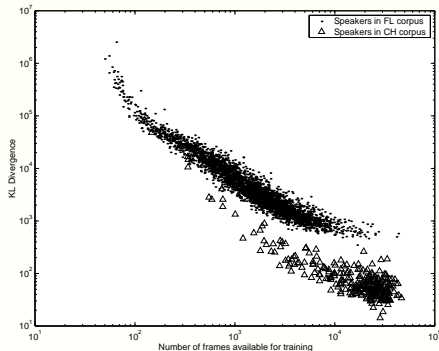
## Distance Measures Between Model Sets

- ▶ **Objective:** Investigate how 'close' are the MAP derived transforms  $T_{(MAP)}$  to the ML derived transforms  $T_{(ML)}$
- ▶ Two sets of parameters:  $\eta_{(MAP)} = (\theta, T_{(MAP)})$  and  $\eta_{(ML)} = (\theta, T_{(ML)})$
- ▶ Use an average KL divergence between pairs of Gaussians on a per mixture component level, defined as

$$D(T_{(MAP)}, T_{(ML)}) = \frac{1}{\sum_{s=1}^S M(s)} \sum_{s=1}^S \sum_{m=1}^{M(s)} D_{KL}(q(\zeta|s, m; T_{(MAP)}), q(\zeta|s, m; T_{(ML)}))$$

- ▶  $M(s)$  is the number of mixture components in state  $s$

## Average Kullback-Leibler divergence $D(T_{(MAP)}^k, T_{(ML)}^k)$



- ▶ KL divergence  $D(T_{(MAP)}^k, T_{(ML)}^k)$  is 'on-the-average' inversely proportional to the amount of the training data

## Speaker Adaptive Training on Normalized Out-of-Domain Acoustic Data

Data Sources & Normalization				Character Error Rate (%) on CF	
CF	CH	FL	#transforms	SI+MLLR	SAT+MLLR
I	I	I		56.6	55.6
I	T	T	1 per corpus	55.5	54.6

- ▶ We compare the performance of SAT acoustic models trained over:
  - ▶ Unnormalized acoustic data
  - ▶ An in-domain training set created by transforming the out-of-domain corpora prior to SAT.
- ▶ Gains from SAT and cross-corpus normalization are almost exactly additive, and are thus capturing complementary influences
- ▶ Results show that SAT can be further improved by 1.0% if we first compensate for the cross-corpus differences across the training sets.

## Cross-Corpus Normalization Summary

- ▶ Simple attempts to add out-of-domain data degrade performance
- ▶ Developed acoustic normalization procedure for enlarging acoustic training set with out-of-domain data, using techniques developed for speaker normalization in training
- ▶ Experimental results show that cross-corpus acoustic normalization makes it possible to combine severely mismatched corpora to increase the amount of available training data
- ▶ Cross-domain normalization can also improve Speaker Adaptive Training.
  - ▶ Experimental results show that performing SAT over cross-corpus normalized data effectively doubles the gains obtained from SAT alone on this corpus.
  - ▶ Cross-corpus normalization and SAT procedures yield additive improvement.

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- ▶ Developed a novel estimation procedure that finds Discriminative Linear Transforms (DLLT) jointly with MMI for feature normalization
  - ▶ **Result:** DLLT provides a discriminative estimation framework that outperforms its Maximum Likelihood counterpart (MLLT)
- ▶ Developed a structural maximum a posteriori estimation framework for feature-space transforms.
  - ▶ **Result:** MAP estimation of linear transforms provides improved performance relative to ML estimation procedures when data sparseness was encountered.
- ▶ Described a cross-corpus acoustic normalization procedure for enlarging an ASR acoustic training set with out-of-domain acoustic data.
  - ▶ **Result:** Improve performance by adding a severely mismatched corpus that otherwise leads to performance degradation.
  - ▶ **Result:** Cross-corpus normalization can also improve Speaker Adaptive Training (SAT).
    - ▶ Performing SAT over cross-corpus normalized data yields additive improvement.

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- ▶ **Combination of discriminative training within a Bayesian formalism**
  - ▶ To improve MMI estimates, based on insufficient data, incorporate prior information about the transformation parameters into the discriminative training framework
- ▶ **Controlling the contribution of out-of-domain data**
  - ▶ Linear transforms attempt to capture non-linguistic acoustic variations between heterogeneous collections.
  - ▶ Mismatch between acoustic sources may be nonlinear
  - ▶ As an alternative, a weighted version of the classical ML estimation framework can be used
  - ▶ We expect the primary challenge to be
    1. the identification of the out-of-domain data that needs to be downweighted
    2. the estimation of the corresponding weights
  - ▶ One possibility is to use a distance that measures the deviation of the normalized observation from the current estimated model